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Issues on orientifolds: on the brane construction of gauge theories with $\mathrm{SO}(2n)$ global symmetry

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Abstract: We discuss issues related to orientifolds and the brane realization for gauge theories with orthogonal and symplectic groups. We specifically discuss the case of theories with (hidden) global $\mathrm{SO}(2n)$ symmetry, from three to six dimensions. We analyze mirror symmetry for three dimensional N=4 gauge theories, Brane Box Models and six-dimensional gauge theories. We also discuss the issue of T-duality for D_n space-time singularities. Stuck D branes on ON^0 planes play an interesting role.

Keywords: Field Theories in Higher Dimensions, Field Theories in Lower Dimensions, Duality in Gauge Field Theories, Brane Dynamics in Gauge Theories.

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1. Introduction

This paper discusses issues related to orientifolds. The motivation which led to this project was to understand various strong coupling behaviour of orientifold planes. These planes play an important role in understanding the non-perturbative properties of string theories. They can also be used for realizing and studying, using branes, gauge theories with orthogonal and symplectic gauge groups; in this perspective, the dictionary for translating different gauge theories into brane configurations is still under construction. There are several types of orientifold planes. For most of them, we know, at least classically, what is the gauge theory realized on branes living in the presence of these orientifolds. However, as will be discussed below, there are cases (the somehow exotic plane[1] $\hat{O}$ or the ON planes acting on NS-branes) in which the entries in the dictionary are still mysterious. Moreover, even when everything is understood at the classical level, the strong coupling behaviour of these configurations is not well understood; this prevents us from extracting non-perturbative results from the brane configuration for the corresponding gauge theory. Unfortunately, this lack of understanding is common to many different orientifold configurations.

In this paper, we make a first step in the direction of better understanding these configurations and we analyze a class of theories, with various supersymmetries and in various dimensions, with global $\mathrm{SO}(2n)$ symmetry. The issues which will be discussed here in more detail are:

- Mirror Symmetry in three dimensions.
- Various constructions of Brane Box Models involving orientifold planes.
- Some issues in six dimensional theories.
- More constructions of Finite four dimensional gauge theories using branes.

All the theories presented have in common a D_n type global symmetry. Since, besides the use of an orientifold, theories with D_n global symmetry can be realized by engineering space-time singularities, there is a natural question that arises in relation to the models discussed in this paper and that we will discuss in some detail:

- What happens to a D_n space-time singularity when we perform a T-duality?

The answer for this question in the case of a A_k singularity, which first appeared in [1], is extensively discussed in the literature, and here we present the D_n case. The understanding of the behaviour of singularities under T-duality helps in relating (and therefore in better understanding) different approaches with branes to the same gauge theory. For each of the theories considered in this paper, we will discuss the fate of the configurations under T-duality and the relation with different approaches in the literature.

Let us briefly summarize what is known about the orientifold zoo.

There are several types of orientifold planes. The most familiar and most discussed one is the Op plane which carries negative RR charge, that is in units in which the charge of a Dp-brane is positive (we will take the charge of a physical brane to be $+1$ in this paper). The charge of this Op plane is -2^{p-5}. A collection of n physical Dp-branes located at the Op plane give rise to a D_n gauge theory on the world

[1] See the next few paragraphs for a discussion on this object.

volume of the Dp-branes, that is an enhanced SO$(2n)$ gauge theory. Henceforth this orientifold will be called Op^-, in short. If there are 2^{p-5} physical Dp branes located on top of an Op^- plane, we get a special case in which the RR charge of this object is zero. The corresponding RR field has no sources coming from such an object which makes it special and leads to interesting phenomena in various dimensions and supersymmetries. For a recent discussion for the case of $O6$ planes see [2]. We will call this object Op^0.

Another type of orientifold is also fairly discussed in the literature and first appeared in [3]. This orientifold plane carries an opposite charge with respect to the Op^- plane, $+2^{p-5}$, and will be called Op^+. A collection of n physical Dp-branes sitting on top of this orientifold plane give rise to an enhanced C_n gauge theory on the world volume of the Dp-branes, that is a Sp(n) gauge theory.

With this realization of gauge theories, the natural question which arises is what is the realization of B_n type gauge theories. (We remind that A_n gauge theories are given by collecting $n+1$ branes on top of each other, with no presence of an orientifold plane). The answer to this question is also well known. If one puts a stuck Dp-brane on the Op^- plane and, in addition, n physical branes, one gets the desired construction. The charge of a stuck Dp-brane is $+1/2$, this means that the charge of the orientifold with a stuck Dp-brane on it is $1/2 - 2^{p-5}$. Henceforth, this orientifold plane will be called $\widetilde{Op}$. Aspects of this orientifold plane were recently discussed in [4].

A last type of orientifold plane is the less familiar object out of all other. This orientifold plane was discussed in various papers [5, 6, 7, 4] where the special cases of $p = 3$ and $p = 4$ were discussed. T-duality suggests the existence of this orientifold plane for any p and this issue will be discussed elsewhere.[2] This orientifold plane will be called $\widehat{Op}$. A universal charge formula for this orientifold plane is not clear at the moment.

Once we accept the existence of these types of orientifold planes, the next natural question which comes to mind is what is the strong coupling behavior of such planes? In many cases the answer appears in the literature. One notable work on this issue is of Sen, [8] where he gives a description of strong coupling behaviour of Op^- planes for various values of p. Less is known about the other types of orientifolds.

A special case is $p = 5$. We ask what is the strong coupling of an $O5$ plane. We will call such an object an ON plane which comes from the fact that S-duality of Type IIB implies that this orientifold carries magnetic charge with respect to the NS two-form of Type IIB. T-duality also implies that there will be a similar object in Type IIA theory, that is an ON plane which carries a NS two-form charge. As for the Op planes, we assume that there are four types of such ob-

[2]It is interesting to look at configurations with orientifold planes and stuck 5 branes. The nature of the orientifold plane changes as one crosses the 5 branes and strong coupling duals of such configurations lead to interesting results.

jects which will be denoted by $ON^-, ON^+, \widetilde{ON}, \widehat{ON}$. The charges of these objects are -1 for ON^-, $+1$ for ON^+, $+1/2$ for $\widetilde{ON}$ and not clear for $\widehat{ON}$. Similar to the Op^0 object, we can have a configuration with a physical NS brane sitting on top of an ON^- plane. This configuration carries no NS two-form charge and will be denoted ON^0. Like the Op^0 objects the absence of NS two-form charge leads to interesting field theory consequences which makes this configuration special.

Another set of questions which arise in the presence of such orientifolds is what is the gauge theory which lives on the world volume of Dp-brane which have some directions which are not parallel to the orientifold planes. As an example, we would like to know what is the world volume gauge theory of a collection of D4-branes which are sitting transverse to an $\widetilde{ON}$ plane in a supersymmetric fashion. All combinations of this type are of natural interest.

In this paper we will mainly focus on theories with SO($2n$) global symmetry. The recent understanding of the properties of the ON^0 plane [9, 10] helps us in extending previous analysis of mirror symmetry in three dimensions [11, 12], Brane Box Models [13, 14, 15] and six-dimensional theories [16, 17, 18] to the case with D_n symmetry. Essentially, this paper extends what was done in the above mentioned papers from the A_k series to the D_n series. In this context, it is of great importance to understand what happens to a D_n ALE space under T-duality. This issue has its own importance even besides its use for realizing gauge theories with branes and it will be discussed in detail in this paper. It is natural to ask about theories with B_n and C_n global symmetry. This is related to the understanding of strong coupling behavior of $\widetilde{Op}$ and Op^+ planes. This will be discusses elsewere.

The paper is organized as follows. In section 2 we review and extend the analysis of the ON^0 plane properties. We extensively discuss the behaviour of D-brane probes near the plane. This section contains the technical tools that will be needed for all the examples in this paper. The subsection 2.2 is more technical; since the results in this subsection will be used only for a particular pair of mirror theories and for six-dimensional theories, the reader not interested in these topics may skip this subsection. In section 3 we discuss the T-duality for an ALE space of type D_n. In the following sections we discuss various examples of gauge theories in various dimensions. In section 4 we present several examples of mirror pairs in N=4 three dimensional gauge theories. In section 5 examples of Brane Box Models and finite N=1 gauge theories in four dimensions. In section 6 examples of six-dimensional superconformal fixed points and small instanton theories. We tried to keep each different section as self-contained as possible, in such a way that the reader only interested in a particular class of gauge theories or only interested in T-duality for ALE spaces may skip what does not interest him. For each class of gauge theories we explicitly discuss T-dual descriptions and try to make contact with different approaches.

2. The ON^0 plane

A particularly important object for our purpose is the plane ON^0, obtained as a superposition of an ON^- plane and a physical NS brane. It arises both as the strong coupling limit of $O5^-$ planes and as an essential ingredient in the T-duality for D singularities.

Its importance is increased by the fact that it has a perturbative description [9]. The ON^0 plane can be considered as the fixed plane of the perturbative $(-1)^{F_L}R$ projection in a Type II string theory. This orbifold projection is the combination of a space-time Z_2 inversion in four directions, R, and the operator $(-1)^{F_L}$, the left-handed space-time fermion number. There are fields living on the world-volume of ON^0. They are the twisted states of the orbifold projection, which, both in Type IIA and Type IIB, have the same massless field content as a NS-brane.[3]

ON^0 acts as a sort of *orientifold* projection for NS-branes. $n-1$ physical NS-branes on top of the ON^0 plane realize an SO$(2n)$ symmetry; this is the standard gauge symmetry of the system of NS-branes in Type IIB, or, the non-Abelian symmetry acting on the tensionless strings of the $(2,0)$ theory in Type IIA. Notice that, in this description, one of the Cartan generators of SO$(2n)$ is not associated with the position of one of the NS branes but rather with the twisted states of the orbifold projection.[4]

There are two properties of ON^0 that make it special. They also explain the proposed SO$(2n)$ symmetry:

- S duality. In the Type IIB context, $n-1$ NS-branes near ON^0 plane give the S-dual realization of $(1,1)$ SO$(2n)$ six-dimensional theories with n physical D5-branes on top of an orientifold plane. S-duality indeed maps D5-branes into NS-branes, and $O5$ planes into ON planes. ON^0 is the strong coupling limit of the corresponding $O5^0$ plane, the superposition of an $O5^-$ orientifold and a physical D5 brane [9]. As noticed in [9], the $O5^0$ plane, as opposed to $O5^-$ supports fields on its world-volume and it is not charged under the RR six-form; it has therefore the right characteristic for being the strong coupling limit of the orbifold fixed plane.

- T duality. ON^0 is also connected to the T-dual of a Type IIA D_n singularity. The proposal is that a T-duality along one of the directions of a Type IIA D_n singularity is the Type IIB orbifold $R^4/(-1)^{F_L}R$ in the presence of $n-1$

[3]The twisted states are vector supermultiplets of a $(1,1)$ six-dimensional theory in Type IIB and $(2,0)$ tensor supermultiplets in Type IIA.

[4]It is interesting to consider the brane realization of the BPS spectrum of these theories. The W-bosons are realized by D-strings, and the magnetically charged two-branes are realized by D3 branes. Near the ON^0, the realization is different than the usual one near Op planes. See figure 2 for some of these states.

physical NS-branes. The gauge symmetry is in both cases SO$(2n)$. The n twisted states of the D_n Type IIA singularity, which are $(1,1)$ six-dimensional vector multiplets, are mapped to the $n-1$ multiplets living on the NS-branes in Type IIB plus the extra multiplet living on the plane ON^0. This proposal can be motivated by representing a T-duality as a combination of two strong-weak coupling dualities, as depicted in figure 1. A background with a D_n singularity can be described as a particular decompactification limit of Type IIA on a singular $K3$, which is dual to the Type I$'$ string theory on T^4. In the relevant decompactification limit, the dual theory can be represented as Type IIB on $R^4/\Omega R$. In this dual description, the non-perturbative enhanced SO$(2n)$ gauge symmetry of Type IIA is described by n physical D5-branes near an $O5^-$ orientifold plane. Another S-duality now brings us back to the Type IIB orbifold $R^4/(-1)^{F_L}R$ in the presence of NS-branes. Similar arguments apply for Type IIB singularities and Type IIA ON planes and NS branes.

The importance of ON^0 both as the strong coupling of $O5$ plane and as an ingredient in the T-dual description of D_n singularities is manifest in figure 1. As strong coupling of an $O5^0$ plane, ON^0 plays an important role in the study of mirror symmetry in three dimensional $N=4$ gauge theories. As ingredient in the T-dual description of D_n singularities, it plays an important role in better understanding several brane configurations, varying from Brane Box Models to six-dimensional theories.

But, before constructing explicit examples, we need to explain what is the behavior of D-branes in the presence of an ON^0 plane.

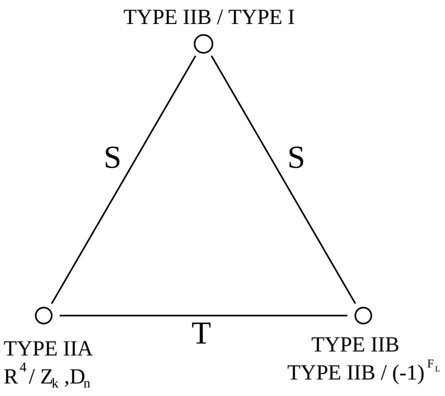

Figure 1: A graph consisting of commuting T and S dualities. The T-duality between Type IIA at an orbifold singularity and Type IIB with NS-branes can be represented as two successive strong-weak coupling dualities.

2.1 D-branes in the presence of an ON^0 plane

Consider a Dp-brane ($p \leq 6$) transverse to the ON^0 plane, or the Type II orbifold $R^4/(-1)^{F_L}R$. Let us take, for definiteness, the R inversion acting on the coordinates $6,7,8,9$.

The crucial point is that the twisted sector of the $(-1)^{F_L}R$ orbifold has the same field content as a NS-brane and, therefore, a D-brane can end on it. This means that the D-brane is a source for the fields living at the fixed plane[5] As noticed in [9], D-branes ending on the orbifold fixed plane may have both positive and negative charge

[5]Notice that this is different from the standard configuration in which a D-brane crosses an orientifold plane. Since the orientifold plane does not carry any field on its world-volume, the D-brane is not really ending on it.

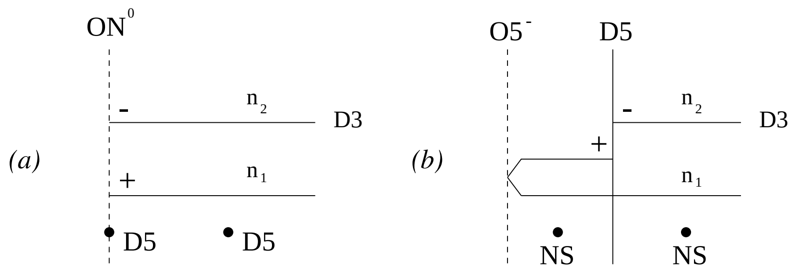

Figure 2: Two useful ways of thinking of ON^0 planes. In the S-dual description (b) the different signs for the charges are explained by considering branes ending on the D5 brane from the right or from the left. We may also put extra D5-branes (and NS-branes in the S-dual configuration) for future reference. In (b), we put two kinds of D5-branes (represented as dots). One of them is bound to the ON^0 plane; in the S-dual picture (b) it has the interpretation of a NS-branes which lives in between $O5^-$ plane and the D5-brane.

under the twisted fields; moreover, configurations of parallel D-branes with different charges are supersymmetric [9, 10]. The reader should not confuse these negatively charged D-branes with anti D-branes. Anti D-branes have a negative charge under the ten-dimensional RR-forms. In this paper we always consider D-branes. Our branes are charged both under the ten-dimensional RR-form (with positive charge) and under the six-dimensional twisted sector form (with positive or negative charge).

The existence of different charges and the fact that supersymmetry is preserved can be easily understood in the case $p = 3$ going to the S-dual configuration [9]. The result for general p follows from T duality. Consider figure 2. In the S-dual picture, the orbifold plane is represented by an orientifold $O5^-$ plane and a physical D5-brane which can be moved away from the orientifold point. The twisted sector of the orbifold point are mapped to the fields on the D5-brane. D3-branes ending on the orbifold plane are now ending on the D5-brane. Their charge under the D5-fields has a different sign according to whether they end to the left or to the right of the D5-brane. We can identify the positively charged branes ending on the orbifold plane with the D3-branes ending on the D5-brane from the right, while the negatively charged branes with D3-branes coming from the right infinity, going straight to the orientifold, coming back and ending on the D5-brane from the left. This configuration is manifestly supersymmetric.

n Dp-branes ending on the fixed plane, all of them with the same charge, have a world-volume theory with eight supersymmetries consisting of a U(n) gauge theory. There are no matter fields, since the hyper-multiplets corresponding to the fluctuations transverse to the orbifold plane are projected out by the action of R. The rules for projecting the open string spectrum in the case with both positive and negative charges were found in [9, 10] by using a boundary-state method. The rule is that, if there are n_1 Dp-branes with positive charge and n_2 Dp-branes with negative charge

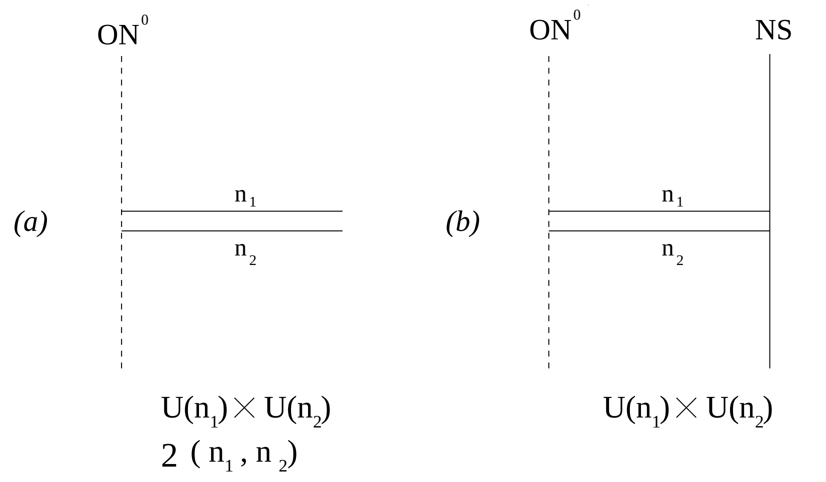

Figure 3: Two sets of D-branes (n_1 with positive charge and n_2 with negative charge) ending on ON^0 plane. The resulting gauge theory and matter fields are indicated below the figure. In (a) there are two hypermultiplets in the bi-fundamental representation of the two gauge groups. These hypermultiplets parametrize fluctuations transverse to the orbifold plane. In figure (b), the two hypermultiplet is projected out by the presence of the NS brane.

ending on the fixed plane, $(-1)^{F_L}R$ acts on the open string Chan-Paton factor as the conjugation by a diagonal matrix with n_1 entries equal to $+1$ and n_2 equal to -1. This means that the gauge fields (and their superpartners) coming from open strings connecting D-branes with different charge are projected out and the gauge group is $\mathrm{U}(n_1) \times \mathrm{U}(n_2)$. The hyper-multiplets corresponding to open strings with both ends on D-branes with the same charge are projected out being odd under R, but the ones associated with *mixed* open strings have an extra minus sign and survive giving matter fields in the bi-fundamental (n_1, n_2) representation (figure 3). The same result can be obtained by looking at the S-dual configuration described in figure 2.

In this paper we will need configurations with more NS-branes and with other Dq-branes. They are depicted in figure 4a. When there are both Dp and Dq-brane, it is better to use pictorial conventions in which Dp-branes are horizontal lines, NS-branes are points and Dq-branes or Oq planes are vertical lines. Here q and the directions in which the Dq-brane extend are chosen in such a way that preserves supersymmetry in the presence of Dp-branes; the number of directions of type D-N in the open strings connecting Dp and Dq-branes must be 4. Dp-Dq configurations considered in this paper are the D3-D5 systems relevant for mirror symmetry in three dimensions, the D5-D7 systems for Brane Box Models, and the D6-D8 systems for the construction of six-dimensional theories.[6] The Dq-branes serve for introducing matter fields in the fundamental representation of the various gauge groups. The world-volume theory

[6]See section 2.2 for the explicit directions for the Dp and Dq branes.

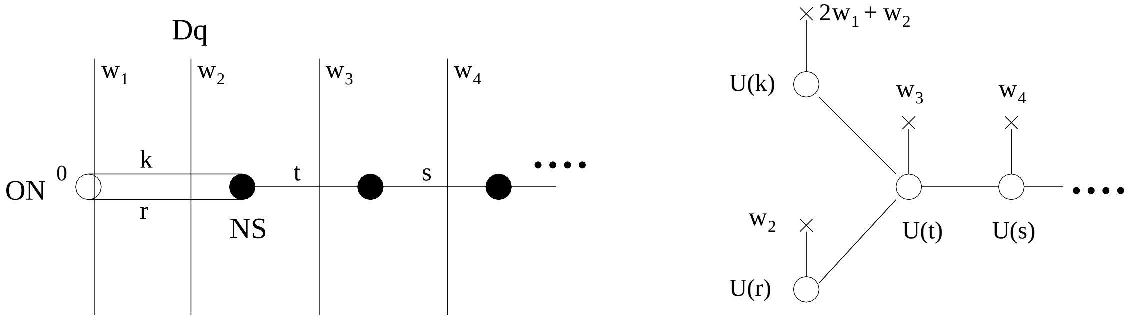

Figure 4: The gauge theory can be read from the quiver diagram on the right. Nodes represent the gauge group factors, links represent bi-fundamental matter fields and external lines represent fields in the fundamental representation of the corresponding gauge group.

obtained in this way can be conveniently encoded in a quiver diagram, where nodes represent the gauge group factors, links represent bi-fundamental matter fields and external lines represent fields in the fundamental representation of the corresponding gauge group; the quiver is depicted in figure 4*b*.

The curious splitting of the Dq-branes near ON^0 requires some explanation. The w_1 Dq-branes in figure 4*a* are *bound* to the ON^0 plane. The existence of bound branes can be easily understood in the particular case of $p = 3$ and $q = 5$; in the S-dual picture of figure 2*b*, they are described by the NS-branes which live in between $O5^-$ and the D5-brane. It is obvious from figure 2 that they only contribute flavors to the U(p) gauge group. The w_2 Dq-branes are instead mapped to NS-branes which live to the right of the D5-branes; they contribute flavors to both the U(p) and U(q) gauge groups.

The global symmetry for a generic gauge factor is U(w_i). The gauge theory has a $\mathrm{U}(2w_1) \times \mathrm{U}(w_2)$ symmetry for the factors associated with branes near ON^0. Notice that only a subgroup $\mathrm{USp}(2w_1) \times \mathrm{U}(w_2)$ of this global symmetry is *manifestly* realized in the brane picture. Further arguments in favor of this symmetry will be discussed in section 4.

This kind of quiver theories naturally represent a small U(w) instanton sitting on a D_n ALE space [19, 20]; here w is the total number of Dq-branes. The configuration with NS-branes is the T dual of the one considered in [19], as will be discussed in section 3. The form of the theory is the same for every value of p.

2.2 Including orientifold planes

In this subsection we further complicate our life by introducing an orientifold plane in addition to ON^0. This configuration will be used in section 4.4 for a particular class of mirror pairs in three dimensions, and in section 6 for discussing six-dimensional theories. The reader not interested in these examples can skip this subsection.

We now discuss the behavior of Dp-branes near an ON^0 plane when we also introduce an Oq^- plane to the picture.

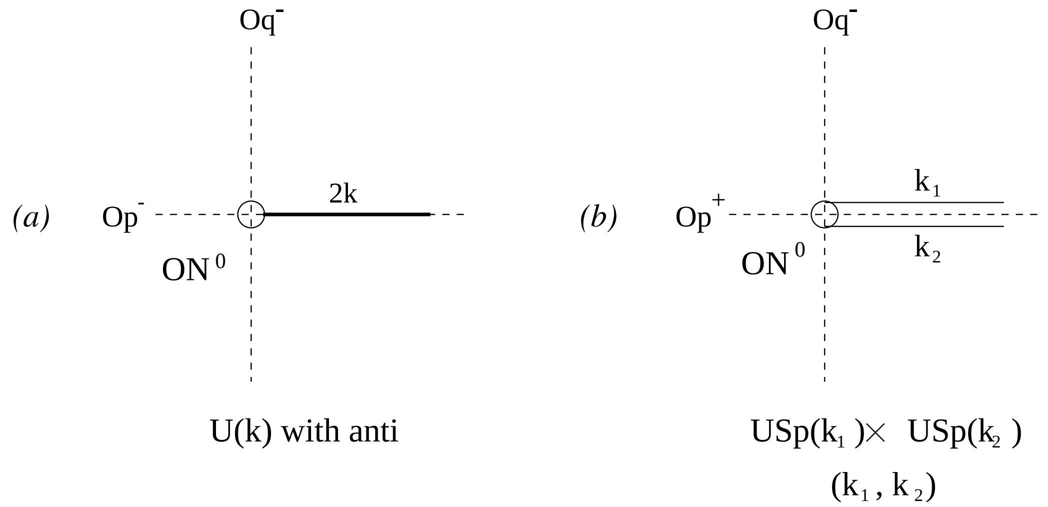

Figure 5: In the case 'without vector structure' (figure (a)), the states living on ON^0 and responsible for absorbing the charge of Dp-branes are projected out by Ω. Therefore there is only one type of Dp-brane, living at the intersection of Oq^- and Op^- planes. This is the theory discussed in [3]. In the case 'with vector structure' (figure (b)), the surviving states living on ON^0 allow and require the existence of two kinds of Dp-branes. Each set of branes supports a USp(k) group due to the existence of an Op^+ plane.

In this paper, we consider ON^0 as a perturbative orbifold projection; in this, it differs very much from its natural partners, the NS-branes, which are solitonic objects. The Oq^- and the ON^0 plane combine into a $Z_2 \times Z_2$ orbifold/orientifold projection, generated by ΩR_q and $(-1)^{F_L} R_{6789}$, where R_q represents a Z_2 inversion in all the coordinates transverse to the Oq plane. We are using notation in which the ON^0 plane extends along (012345) and the Dp-branes are stretched along $(0, 1, \ldots, p-1)$ and are possibly finite along x_6. The Oq plane extends over the coordinates $(0, 1, \ldots, p-1)$ and $(7, 8, 9)$. We will denote R_{6789}, in short, R.

We now discuss what kind of theory is realized on Dp-branes in this situation. The reader not interested in technical details may want to skip the two following paragraphs dealing with tadpoles, projections and all that, and look at figure 5. In the following we present two derivations for the massless fields in this configuration.

First derivation. Every generator of the orbifold/orientifold projection acts on the Chan-Paton factors of the D-branes according to the rules in [3, 19]. Under a T-duality in the directions $p, \ldots, 5$ and 6 , the factor $(-1)^{F_L}$ disappears [21], and we recover a non-compact version of the original model in [3]. As widely discussed in the literature [19, 22, 23, 24, 25], there are essentially two different consistent models. They differ in the perturbative definition of the Ω projection on the closed string twisted states and in the action of Ω on the Chan-Paton factors. Geometrically, they are distinguished by the type of SO$(2m)$ bundle that can be defined on a space with a Z_2 singularity [22]. This SO$(2m)$ bundle is realized on the world-volume of the Dq-branes. A bundle defined on a space with a Z_2 singularity may admit vector structure, or not. The original example in [3] does not admit vector structure. Modifications that admit vector structure were constructed in [26, 22, 23, 24, 25]. The difference between the two cases is encoded in the following relation between the matrices that act on the Chan-Paton factors [19, 23, 24, 25]:

$$\gamma_{[(-1)^{F_L} R]} = \pm \gamma_{\Omega R_q} \gamma^T_{[(-1)^{F_L} R]} \gamma^{-1}_{\Omega R_q} \qquad \text{with/without vector structure} \tag{2.1}$$

Since we put an Oq^- plane, which by definition determines an SO$(2m)$ symmetry on the world-volume of Dq-branes, $\gamma_{\Omega R_q}$ acts as the identity matrix on Dq-branes [3]. The rule that the symmetry of the matrix that projects Dp-branes is opposite to the one that projects Dq-branes [3] is then enough to (almost) uniquely fix the projection matrices, modulo an irrelevant change of basis. The result is that, in the case without vector structure, the world-volume theory for $2N$ Dp-branes is U(N) with antisymmetric matter fields, and, in the case with vector structure, is USp$(2N)\times$ USp$(2N)$ with a bi-fundamental matter field. The sign in equation (2.1) is related to a global sign in the action of Ω on the closed string twisted sector. The projection by ΩR_q breaks the Lorentz-invariance of the six-dimensional theory of the twisted states. To avoid confusion, it is better to work in the particular case in which $p = 6$, in which this Lorentz invariance is not broken; this is the configuration of D6 and D8 branes relevant for the study of six-dimensional theories. The result for generic Dp and Dq-branes near ON^0 follows from T-duality. The twisted sector provides a tensor multiplet and a hyper-multiplet. In the case without vector structure the tensor multiplet is projected out and the hypermultiplet survives [23]. The opposite happens in the case with vector structure. Undoing the T-duality for discussing the case with generic p, we discover that, in the case without vector structure, the twisted state which may absorb the charge of a Dp-brane is projected out by the Ω projection, while in the case with vector structure it survives. The sign in equation (2.1) is also related to the sign of the RR charge of the Op plane that is induced in the theory. It is the standard Op^- plane in the case without vector structure, but it is Op^+ in the case with vector structure [23].

Second derivation. In [17] we exploited a general method for studying the consistent Type II and Type I models, living at Z_n singularities, where differences between the models (presence or absence of vector structure) were manifest in a dual brane description. Locally, the system $ON^0 - Oq$ is indistinguishable from the generic Z_2 orbifold/orientifold, which was studied in [17]. In the next sections, we will add more NS-branes and the methods in [17] will not be sufficient for describing the full system, but, at that time, we should already know how to deal with ON^0. The methods in [17] allow to study the possible Z_2 orbifold/orientifold models in a T dual picture where the singularity is replaced by two NS-branes.

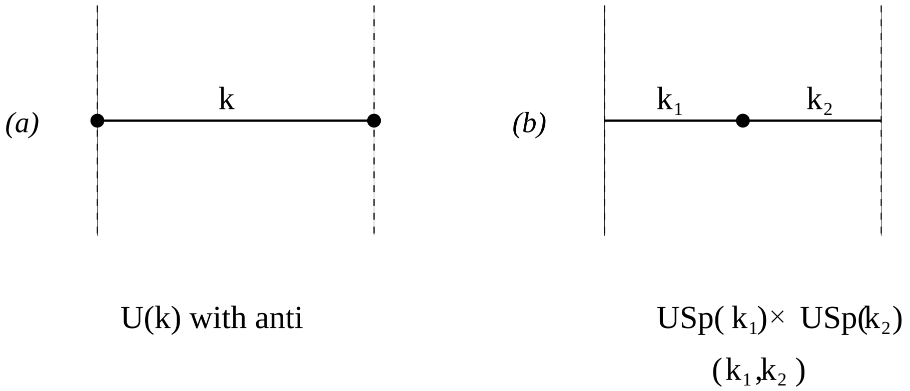

Figure 6: Z_2 orbifold/orientifold models, without (a) or with vector structure (b). Details are exhaustively discussed in [17]. Lines are Dq-branes and points are NS-branes.

To avoid confusion, we again specialize our discussion to the case with $p = 6$; a T-duality gives the result for generic p. The two ways of putting two NS-branes on a circle, respecting the Z_2 symmetry, give two possible models, depicted in figure 6. The gauge group is, in the case without vector structure, U(N) with antisymmetric matter, and, in the case with vector structure, USp(N) $\times$ USp(N) with a bi-fundamental. In the first case, there are no tensor multiplets, while in the second case there is one. Applying a T-duality for recovering the case with generic p, we discover that, in the case without vector structure, the twisted state which may absorb the charge of a Dp-brane is projected out by the Ω projection, while in the case with vector structure it survives.

The results of this long analysis are summarized in figure 5. The results are consistent with a naive extrapolation of the rules discussed in section 2.1 to the case with an orientifold. The crucial point of the previous analysis is that the states associated with ON^0, responsible for the splitting of the branes, survive the orientifold projection only in the case with vector structure. Therefore, we can have two kinds of Dp-branes, and the typical D-quiver splitting only in the case with vector structure. In the case without vector structure, we have only one kind of brane, which is projected according to the original example in [3].

Consider now the addition of NS-branes. This gives the theory depicted in figure 7. We face a lot of small subtleties. The following are the general rules for determining the matter content.

1. At every NS-brane the charge of the Op plane changes [27]. Gauge groups associated to $2k$ Dp-branes near Op^+ are USp$(2k)$, while those associated to k branes near Op^- are SO(k). The only exception occurs when an Op^- crosses ON^0; in this case the gauge group associated to $2k$ Dp-branes is U(k).

2. The bi-fundamental (r, k) fields appearing in figure 5, associated to the branes near ON^0, are absent when the branes end on a NS-brane, since the latter

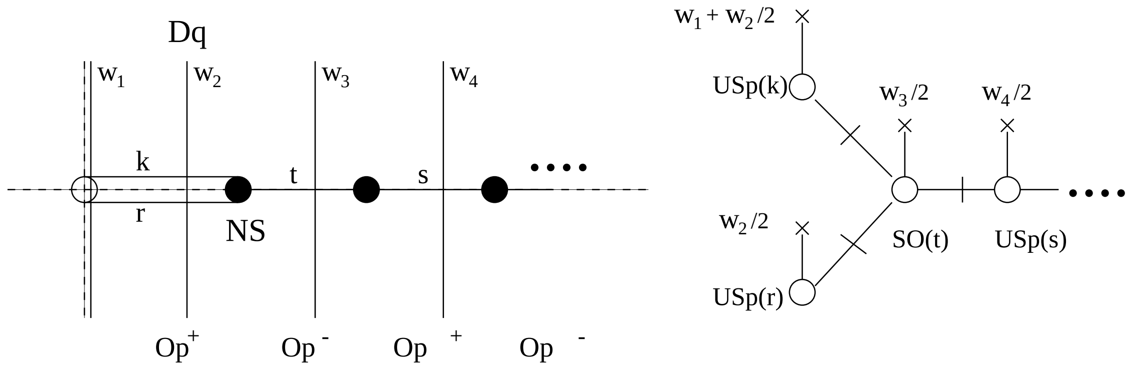

Figure 7: The gauge theory can be read from the quiver diagram on the right. The bar on the links means that they give rise to a half bi-fundamental.

freezes all fluctuations transverse to itself. This is the general case, unless one considers the degenerating examples associated to a singularity of type D_2.

3. There are only half bi-fundamentals for the gauge groups USp $\times$ SO. This is due to the Op projection. This rule does not apply for gauge groups USp $\times$ U or SO $\times$ U.

4. The Dq-branes can be partitioned among the NS-branes and give rise to matter fields in the fundamental representation. w_i Dq-branes located where an Op^+ plane exists give rise to an SO(w_i) global symmetry, while those located where an Op^- plane exists give rise to a USp(w_i) global symmetry. This global symmetry teaches us that $w_i, (i \neq 1, 2)$ Dq-branes give rise to $w_i/2$ flavors. Near ON^0 (w_1 and w_2) there is a further splitting for the Dq-branes. This is analogous to the splitting encountered in section 2.1. The global symmetry is SO$(2w_1) \times$ SO(w_2). The field theory may get a bigger global symmetry SO$(2w_1 + w_2) \times$ SO(w_2) not seen by the branes.

This kind of quiver theories naturally represents small SO(w) instanton on a D_n ALE space [19]. The configuration with NS-branes is the T dual of the one considered in [19], as will be discussed in section 3. The form of the theory is the same for every value of p. In the particular case $p = 6$ the world-volume theory is uniquely specified by the requirement of anomaly cancellation. This will be exhaustively discussed in section 6.

3. T-duality for D_n singularities

In this Section, we discuss in more detail the T-dual of a D_n singularity. It is well known that, in Type II string theory, a T-duality along one of the directions of an ALE space of type A_k transforms it into a Type II flat background with $k + 1$ NS-branes [1]. As already anticipated, the proposal is that the T-dual configuration for an ALE space of type D_n is the Type II orbifold $R^4/(-1)^{F_L}R$ in the presence of $n-1$ physical NS-branes.

In the A_k case, the $k+1$ twisted states of a Z_{k+1} orbifold[7] are mapped to the world-volume fields of the $k+1$ NS-branes.[8] In the D_n case, the NS-branes world-volume fields provide the dual description of only $n-1$ twisted states. The dual of the remaining twisted state comes from the fixed point of $(-1)^{F_L}R$; the twisted sector of $(-1)^{F_L}R$, both in Type IIA and Type IIB, has indeed the same massless field content as a NS-brane.

It is sometimes useful to replace the ALE space with a Taub-NUT geometry (also called KK monopole). Globally the structure of an ALE space is R^4, that of a Taub-NUT space is $R^3 \times S^1$, which makes it more suitable for performing T-dualities. The ALE space can be recovered by sending a parameter (which roughly measures the radius of the circle) to infinity. The T-dual of a D_n KK monopole is the $R^3 \times S^1/(-1)^{F_L}R$ orbifold with $n-2$ NS-branes. Since there are two fixed points of $(-1)^{F_L}R$ along S^1, only $n-2$ NS-branes are now required to match the n twisted states. The T-dual of the D_n orbifold is obtained by sending the radius of the dual S^1 to zero.

The proposal was motivated before using the chain of dualities depicted in figure 1. Related arguments can be found in earlier literature [21, 28], where it was noticed that the perturbative T dual of the singular $K3$ manifold T^4/Z_2 is $T^4/(-1)^{F_L}Z_2$ and a description in terms of NS-branes was proposed.

We can now ask what happens when we introduce D-brane probes in the picture. There is an almost complete dictionary for determining the world-volume theory of D-branes sitting at ALE orbifold singularities [19, 20]. N Dp-branes sitting at a singularity of the form R^4/Γ_G, where G is a simply-laced group, have a world-volume theory that is associated with the extended Dynkin diagram for G, with a gauge factor for each node of the diagram and a bi-fundamental matter field for each link [19]. The gauge group is $\prod \mathrm{U}(n_\mu N)$, where n_μ are the Dynkin indices for the group G. The Higgs branch of these theories, which is the same for all p and is not corrected by quantum effects, is the symmetric product of N copies of the ALE space; this is the brane realization [19] of the well known mathematical construction of the ALE spaces as hyperKähler quotients.

We should be able to see that the world-volume theory of D-branes probe is preserved by the previously discussed T-duality. The analysis of the A_k case is straightforward. N Dp-branes ($p \leq 6$) sitting at a Z_{k+1} singularity have a world-volume theory $\mathrm{U}(N)^{k+1}$ with bi-fundamentals for neighboring $\mathrm{U}(N)$ factors. Deforming the ALE space to a Taub-NUT and performing a T-duality along the S^1, we obtain a

[7]The twisted states are vector supermultiplets of a $(1,1)$ six-dimensional theory in Type IIA and $(2,0)$ tensor supermultiplets in Type IIB.

[8]NS-branes indeed support vector multiplets in Type IIB and tensor multiplets in Type IIA.

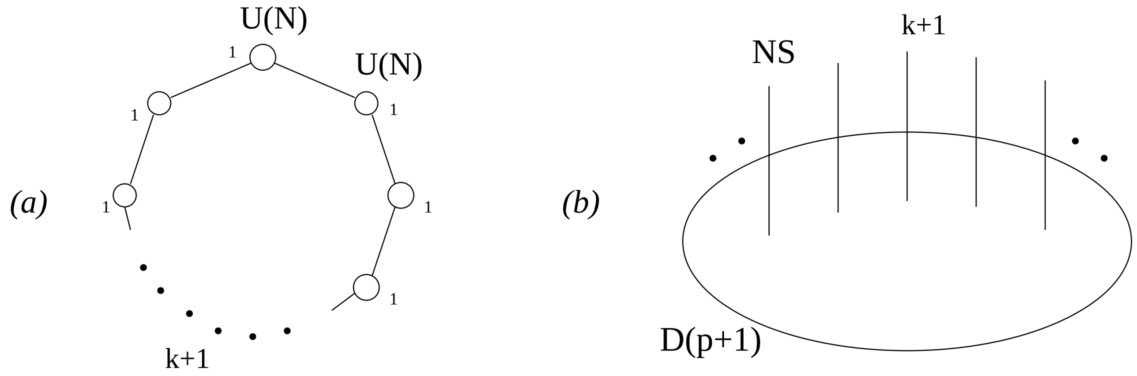

Figure 8: In figure (a), the extended Dynkin diagram for A_k is depicted. The small numbers near each node are the Dynkin indices. Each node is associated with a gauge group factor and each link with a hypermultiplet in the bi-fundamental representation of the corresponding groups. The resulting theory is $\mathrm{U}(N)^{k+1}$ with bi-fundamentals for neighboring $\mathrm{U}(N)$ factors. The same theory can be obtained by considering figure (b), which describes D-branes wrapped around a circle and broken in between $k+1$ NS-branes.

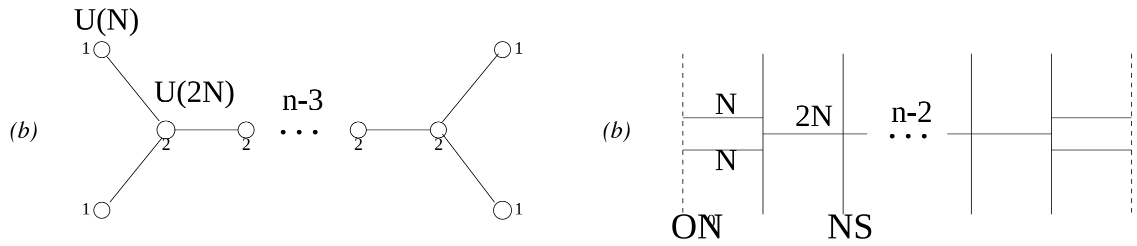

Figure 9: In figure (a), the extended Dynkin diagram for D_n is depicted. The small numbers near each node are the Dynkin indices. Each node is associated with a gauge group factor and each link to a hypermultiplet in the bi-fundamental representation of the corresponding groups. The resulting theory is $\mathrm{U}(N)^2 \times \mathrm{U}(2N)^{n-3} \times \mathrm{U}(N)^2$ with bi-fundamentals for neighboring factors. The same theory can be obtained by considering figure (b), which describes D-branes wrapped around a circle with two fixed points, here depicted as a segment.

configuration of N D$(p+1)$-branes in the presence of $k+1$ NS-branes, configuration that was discussed in [11] and reproduces the same world-volume theory (figure 8).

That the T-duality process gives a consistent result also in the D_n case can be now easily shown using the results in the previous sections. The world-volume theory for $2N$ Dp-branes sitting at a D_n singularity is $\mathrm{U}(N)^2 \times \mathrm{U}(2N)^{n-3} \times \mathrm{U}(N)^2$ with bi-fundamentals associated with the links of the D_n extended Dynkin diagram (figure 9). After T-duality, we have a set of $2N$ D$(p+1)$-branes wrapped around a circle with two fixed points under $(-1)^{F_L} R$. We can picture the projected circle as a segment. We also have $n-2$ NS-branes. Combining the methods in [11] with those described above, it is straightforward to check that the theory associated to this configuration of branes is the same as that associated to the D_n extended Dynkin diagram.

Configurations in which M Dq-branes are also present are interesting. We are considering a situation in which the Dp-probe is sitting at a point in the ALE space, $q = p + 4$ and the Dq-brane is wrapped on the ALE space. This configuration naturally describes N small U(M) instanton on an ALE space; the Higgs branch of Dp-branes world-volume theories, which is the same for all values of p and is not corrected by quantum effects, is isomorphic to the N instantons moduli space [19]. The case of SO($2M$) instantons is easily obtained by considering an Oq plane. After a T-duality, we obtain a configuration with D($p + 1$), D($q - 1$) and NS-branes. The world-volume theory can be read using the results in sections 2.1 and 2.2 and agrees with the one discussed in [19]. The representation of small instantons sitting at space-time singularities as systems of D and NS branes was discussed in [17] for the case of the A_k ALE spaces. The replacement of space-time singularities with dual smooth backgrounds with NS-branes has a double purpose. First, it allows to have a better control on parameters and moduli of the theory. Second, it provides a very simple and intuitive classifications of the gauge bundle compatible with the singularity; it naturally distinguishes between bundles with or without vector structure and accounts for the breaking to a subgroup due to the singularity. The analysis in [17] can be easily extended to the D_n ALE space using the results in sections 2.1 and 2.2. U(M) instanton theories are associated to the configurations in figure 4 and SO($2M$) instanton theories to the configurations in figure 7; both the figures must be made compact by including a second ON^0 plane. Once again, the disposition of the NS-branes on the segment accounts for the allowed U(M) or SO($2M$) bundles and for the breaking of the gauge group to a subgroup. The extension of the construction in [17] to the D_n series is one of the achievements of this paper, but we do not indulge more on this here, since section 6 will be devoted to the subject. The small U(M) or SO($2M$) instanton at D_n singularities will also appear in the discussion about mirror symmetry.

4. Mirror symmetry

The ON^0 plane was used in [10] to explain three-dimensional mirror symmetry for USp(k) gauge group coupled to a hypermultiplet in the second rank antisymmetric tensor representation and n flavors in the fundamental representation. The candidate mirror theory, associated with the D_n extended Dynkin diagram, was guessed in [29] and demonstrated in [12], using M-theory. However, a description in terms of the construction in [11] was still lacking and, with that, something in our knowledge of the dictionary for translating general gauge theories in terms of brane models was missing. The understanding of the strong coupling of $O5^0$ planes clearly closes this gap [9, 10].

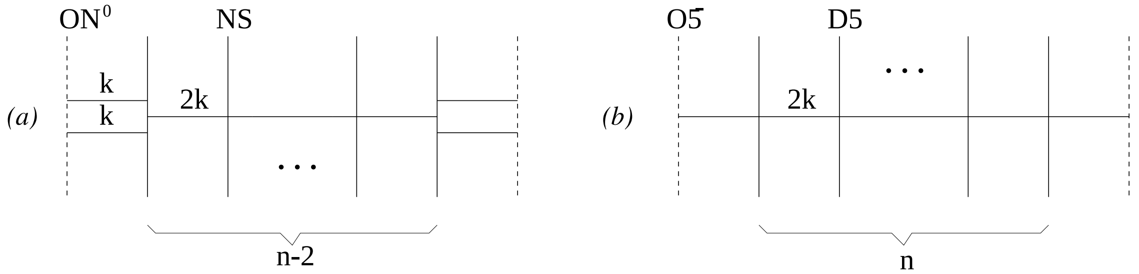

Figure 10: Brane configuration for USp($2k$) with an antisymmetric and n flavors (b) and its mirror (a).

A D_n quiver three-dimensional $N = 4$ gauge theory can be realized as in figure 10a, as extensively discussed in section 3. It is simple, using the rules in [11], to check that the mirror theory is indeed USp($2k$) with an antisymmetric and n flavors [10]. An S-duality transforms NS-branes into D5-branes and the ON^0 plane into a combination of an $O5^-$ plane and a physical D5-brane. As a result, an S-duality transforms the system in figure 10a into a set of D3-branes stretched between two $O5^-$ planes in the presence of n D5-branes, as depicted in figure 10b. This is a fairly standard configuration which realizes the above mentioned USp($2k$) theory. For more details, the reader is referred to [10]. We will shortly show many examples that should make clear this type of construction.

4.1 Examples

The examples that we explicitly discuss are:

- Mirror of USp($2k$) with n flavors in the fundamental representation.
- Mirror of USp($2k_1$) $\times$ USp($2k_2$) with a hypermultiplet in the bi-fundamental representation, n_1 flavors in the fundamental representation of the first group and n_2 flavors in the fundamental representation of the second group.
- Mirror of U($2k$) with one or two matter fields in the two-rank antisymmetric tensor representation and n flavors in the fundamental representation.

Many other examples and generalizations are also outlined.

4.1.1 Mirror of USp($2k$) with n flavors

The mirror theory was found in [30] using different methods; this serves as a check to our method. Later we will study other examples for which there are no known mirrors in the literature. This demonstrates the power of this approach.

The procedure for calculating the mirror theory follows standard steps using the rules in [11]. To move to the Higgs branch of the original theory, one goes to the origin of moduli space, namely to the region in which the D5 branes touch the D3 branes. Once they touch, the D3 branes can split. Figure 11b shows the maximal

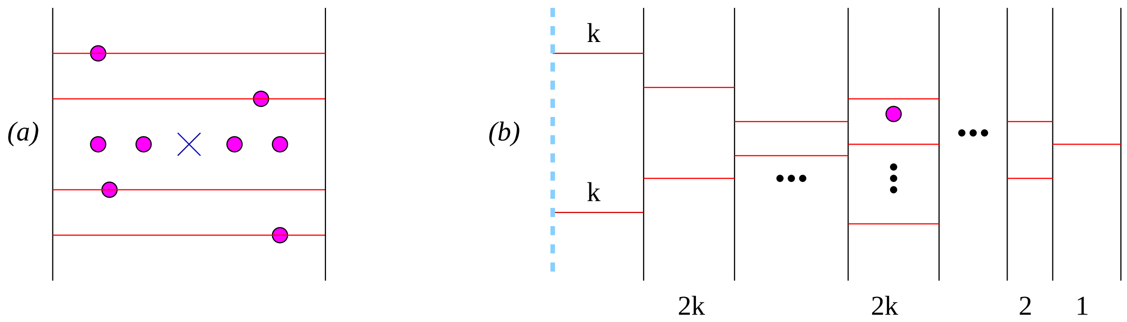

Figure 11: Brane realization for USp($2k$) gauge theory with n flavors in the fundamental representation. Figure (a) shows the coulomb branch of the theory. This figure is drawn on the double cover of the projected space. The X symbol in the middle denotes an $O5^-$ plane. The circles denote D5 branes. The vertical lines represent NS brane and its image, the horizontal lines are D3 branes. Figure (b), drawn only on the physical plane, shows the Higgs branch of this theory, or alternatively the Coulomb branch of the mirror. The branes in the figure are the S-dual of the branes in figure (a), with the same notation for the branes. The numbers below or next to a set of horizontal lines denote the number of D3 branes in between two NS branes.

splitting for the D3 branes. The splitting is done taking into account that S-configurations are not supersymmetric. As a result the NS brane, when located to the right of all branes has D3 brane tails connecting it to $2k$ D5 branes. These tails do not represent massless modes in the field theory. Using the observation that the position of the NS brane is not a relevant parameter for the low energy field theory, we move the NS brane to the left. Whenever it crosses a D5 brane, a D3 brane is removed. After passing $2k$ D5 branes the NS brane is located as in figure 11b. Note that in figure 11b the NS-brane has been denoted as a circle. This before denoted a D5 brane; we have indeed performed an S-duality. At this point we learn that the gauge group in the segment where the circle is located gets an extra hypermultiplet. The final gauge group can be encoded into the quiver diagram depicted in figure 12. The resulting gauge group is

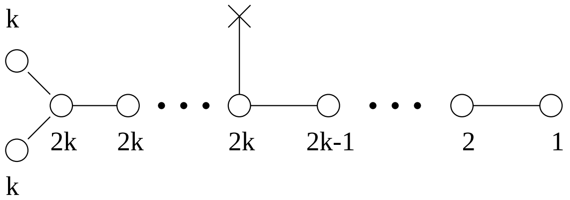

Figure 12: Dynkin diagram for USp($2k$) with n flavors. The numbers next to nodes denote the rank of the gauge factor in the gauge group. Lines denote bi-fundamental hypermultiplets. A line ending on an X denotes a hypermultiplet which is charged with respect to the corresponding node it connects. Later this will be called an external line.

$$\mathrm{U}(k)^2 \times \mathrm{U}(2k)^{n-2k-1} \times \prod_{i=1}^{2k-1} \mathrm{U}(i), \tag{4.1}$$

with matter as in the figure. Note that the first U($2k$) factor has an additional hypermultiplet which is only charged under this group.

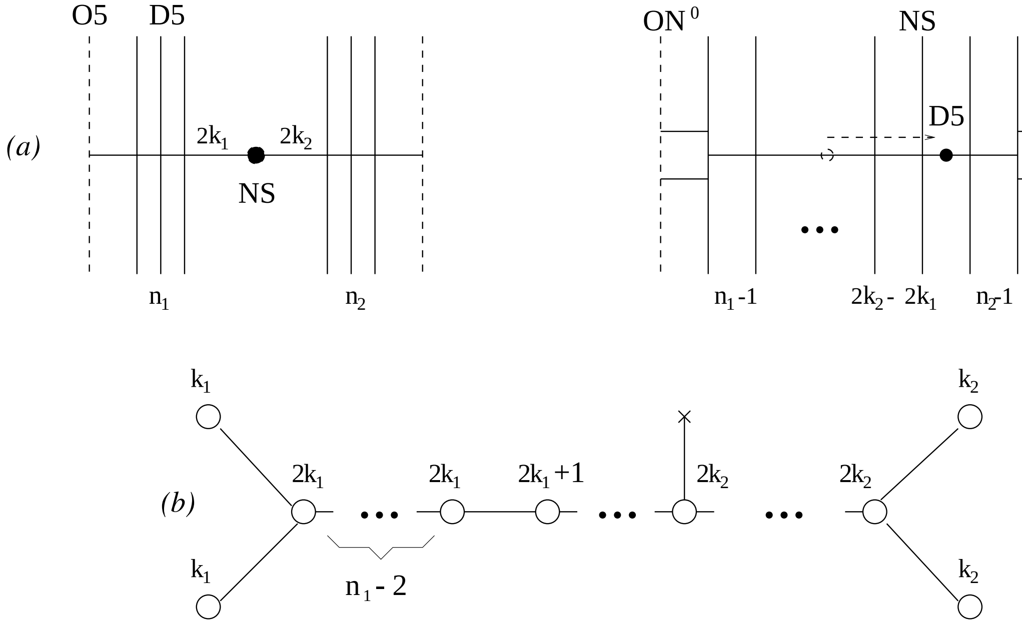

Figure 13: The brane configuration for $\mathrm{USp}(2k_1) \times \mathrm{USp}(2k_2)$ with n_1 flavors in the fundamental representation of the first group, n_2 flavors in the fundamental representation of the second group and a bi-fundamental is shown in figure (a) together with its mirror. The quiver diagram corresponding to the mirror theory is shown in figure (b).

4.1.2 Mirror of $\mathrm{USp}(2k_1) \times \mathrm{USp}(2k_2)$

The next example is the mirror of $\mathrm{USp}(2k_1) \times \mathrm{USp}(2k_2)$ with n_1 flavors in the fundamental representation of the first group, n_2 flavors in the fundamental representation of the second group and a bi-fundamental. The brane configuration is obtained by considering two $O5^-$ planes; it is depicted, with the S-dual configuration, in figure 13a. Notice that we now switched to different notations, which are more suitable for giving a synthetic description of compact models; to avoid confusion we explicitly indicated the type of branes in the picture. The quiver diagram for the mirror theory can be easily computed step-by-step using the rules in [11] and it is shown in figure 13b. The particular case $k_1 = k_2$ is almost trivial to compute; the reader simply needs to exchange NS- and D5-branes and ON^0 and $O5^0$ planes. The case $k_1 \neq k_2$ requires also some attention to non-supersymmetric configurations and to move some of the D5-branes; the needed steps are very similar to the ones discussed in the previous example. The resulting gauge group is, (assuming $k_1 > k_2$ and $n_1 + 2k_2 > 2k_1 + 1$),

$$\mathrm{U}(k_1)^2 \times \mathrm{U}(2k_2)^{n_1-2k_1+2k_2-1} \times \left[\prod_{i=2k_2+1}^{2k_1-1} \mathrm{U}(i)\right] \times \mathrm{U}(2k_2)^{n_2-1} \times \mathrm{U}(k_2)^2 \,.$$

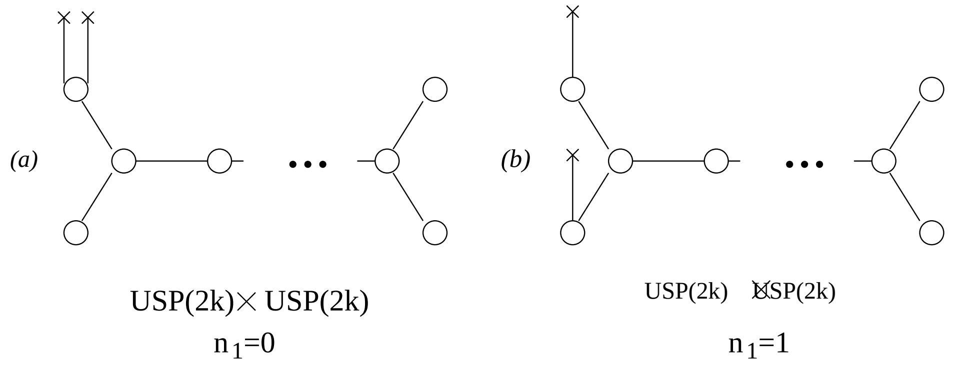

Figure 14: Quiver diagrams of type D with some external lines. The mirror theories are indicated below each graph.

Special attention is required for the cases $n_1 + 2k_2 - 2k_1 = 0, 1$. Without loss of generality, we consider from now on $k_1 = k_2$. For $n_1 = 0, 1$, one should analyze the global symmetry of the three dimensional theory, interpreted as the world volume theory which lives on the NS brane. For generic n_1, the NS brane supports the usual (1,1) supersymmetric six dimensional U(1) multiplet. This is even true for the case $n_1 = 1$. For $n_1 = 0$, we have enhanced symmetry which can be seen before making the mirror operation. The NS brane has a modulus which measures the distance from the $O5^-$ plane. When this distance goes to zero a W-boson, given by D-string stretching between the NS brane and its image under the $O5^-$ plane, becomes massless. The resulting group on the world volume of the NS brane is enhanced to USp(2). More generally, n NS branes next to an $O5^-$ plane have an enhanced USp($2n$) gauge theory on their world volume. It should be noted that Poincaré invariance in six dimensions no longer holds since the $O5^-$ plane breaks it explicitly. The world volume theory of the NS brane is then a six dimensional theory with a point like singularity at three of its coordinates. This breaks half of the supersymmetries in addition to the explicit breaking of six dimensional Poincaré invariance.

The case $n_1 = 0$ becomes even more special when we consider the configuration after making the mirror transformation. The crucial point is that in the S-dual picture the orientifold plane and a D5-brane combine into an ON^0 plane. For $n_1 = 0$, the NS-brane, which in the original configuration lives between the orientifold and the D5-brane whose fate is to combine with it, becomes, after S-duality, a D5-brane that is bound to ON^0. We already discussed this kind of configurations in section 2.1. The enhanced symmetry on the world volume theory of the D5 brane remains, as before the mirror transformation, a USp(2) gauge theory.

Combining this with the symmetry on the world volume of the D5-branes, we get that the global symmetry of the problem for $n_1 = 0$ is USp(2) $\times$ SO($2n_2$). For $n_1 \neq 0$ the global symmetry is U(1) $\times$ SO($2n_1$) $\times$ SO($2n_2$). The knowledge of the global symmetry helps in deriving the quiver diagram corresponding to the mirror theories. The diagrams for $n_1 = 0, 1$ are drawn in figure 14a and 14b.

4.1.3 SU(N_c) with antisymmetric matter and N_f flavors.

We have seen that there are examples with no external lines, corresponding to figure 13, and with two external lines (figure 14). The natural question to ask is how to get a quiver with only one external line. The naive answer will be to get half of the case in figure 14b. Indeed one needs to put a stuck NS brane on the $O5$ plane. This leads to a quiver with one external line as required. What is the field theory of this configuration of branes? Well, the answer is fairly standard. A stuck NS-brane gives a hypermultiplet in an anti-symmetric tensor representation. On the other hand, the gauge theory on the D3 brane still remains a USp gauge theory since it is projected on the other side of the interval. So we get a theory with the same matter content as for the case with no external lines as in figure 13. It is a known phenomenon that the presence of a single NS brane does not change the matter content but may make one of the parameters in the theory manifest as a deformation of the brane configuration.

The brane configurations for SU($2k$) field theories with antisymmetric matter are given in figure 15a. The steps for computing the mirror theories closely parallel those considered in the previous examples. The computation in the compact case is almost trivial; in the non-compact case, one needs to pay attention to non-

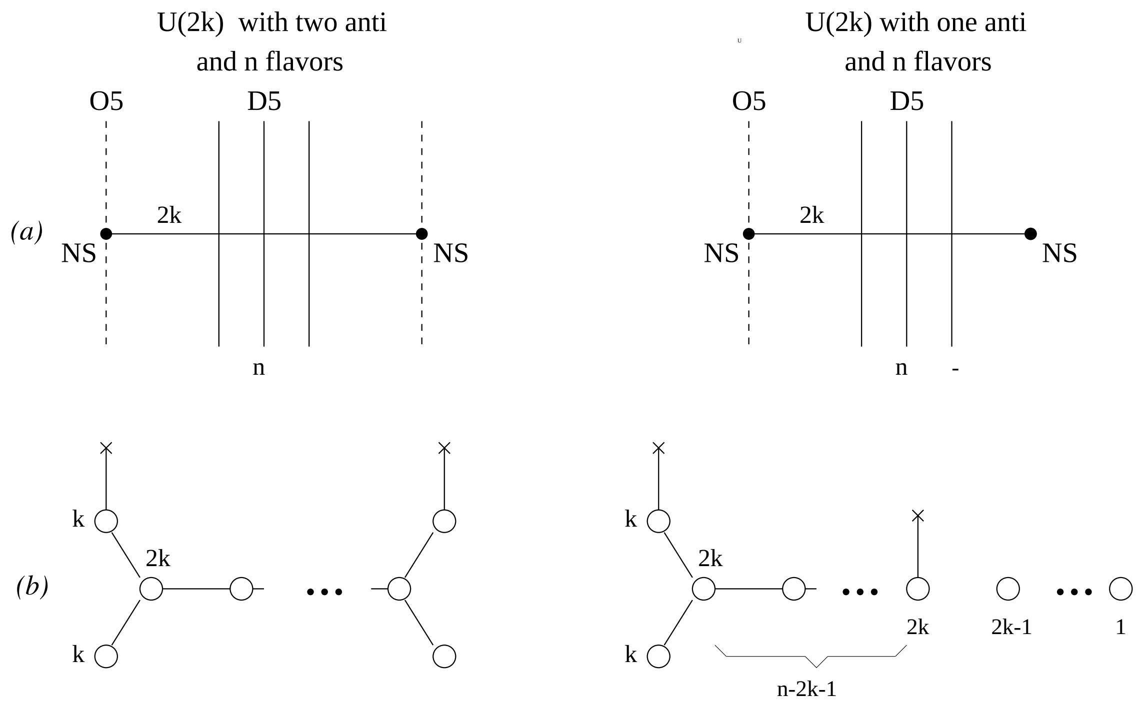

Figure 15: Figure (a): brane realization for SU(N_c) gauge theory with one or two flavors in the antisymmetric representation and N_f flavors in the fundamental representation. Figure (b): the quiver diagrams for the mirror theories.

supersymmetric configurations and move branes around, closely paralleling what is done in section 4.1.1. The results for the mirror theories can be conveniently encoded in the quiver diagrams shown in figure 15*b*. There is no enhanced symmetry associated to the stuck NS branes.

All the considered compact models are associated to a D_n extended Dynkin diagram, decorated with one or two external lines. A convenient classification of these kind of theories is given by looking at the global symmetry; we discussed examples with global symmetry that is a subgroup of $\mathrm{USp}(2) \times \mathrm{SO}(2n)$. The maximally symmetric case is the one in figure 14*a*. In the original theories (and with this we mean the theories realized as configurations of branes with orientifold planes) the $\mathrm{SO}(2n)$ subgroup symmetry is manifestly realized as the symmetry that rotates the flavors transforming in the fundamental representations of the gauge groups; in general, this symmetry is reduced to $\mathrm{SO}(2n_1) \times \mathrm{SO}(2n_2)$. The $\mathrm{USp}(2)$ symmetry instead is not manifest classically, as standard in mirror symmetry [31]; it makes its appearance as an IR symmetry, and, in general, is reduced to $\mathrm{U}(1)$. Alternatively, if one turns on a magnetic gauge coupling, the symmetry is present only at infinite magnetic gauge coupling. In the mirror theories instead the $\mathrm{SO}(2n)$ subgroup symmetry is not manifest classically, while the $\mathrm{USp}(2)$ symmetry (or, generically, its $\mathrm{U}(1)$ subgroup) rotates the flavors corresponding to external lines. These symmetries can be also read from the brane configurations as the symmetries on the world-volume of NS- and D5-branes. All these symmetries are pictorially manifest in the quiver diagram of the mirror theory; $\mathrm{SO}(2n)$ is associated with the D_n Dynkin-diagram form of the graphs and $\mathrm{USp}(2)$ rotates the external lines. The number and the position of the external lines also accounts for the reduction of this symmetry group to a generic $\mathrm{U}(1) \times \mathrm{SO}(2n_1) \times \mathrm{SO}(n_2)$ subgroup.

A natural question is: what is the mirror of the theory corresponding to a D_n quiver diagram with up to k external lines arbitrarily distributed among the nodes? The answer is not difficult to find using the results in the previous sections: we need to consider the case with more NS-branes and make a diagrammatic computation with the rules we discussed. The result will be presented in section 4.3. But, first, let us make some general remarks and try to make contact with different approaches.

4.2 Discussion and relation to other approaches

The step-by-step rules for computing mirror pairs, which we discussed in the previous sections, can be applied to a large variety of theories. In this section, we discuss few general issues about our results and consider examples that have a T-dual description and allow to make contact with different approaches.

All the previous pairs of mirror theories pass the simplest consistency check: the dimension of the Coulomb branch of the theory is equal to the dimension of the Higgs branch of its mirror, and vice versa. The number of masses and FI terms is also consistent with the mirror symmetry expectations, except that in the case of the theories depicted in figure 14. The $\mathrm{USp}(2k)^2$ theory for $n_1 = 0, 1$ indeed has a missing FI term. A similar phenomenon was encountered in [12] and also in [10], in the analysis of D_2 and D_3 quiver diagrams. A possible resolution of this paradox is that the $\mathrm{USp}(2k)^2$ theory has a hidden FI parameter which is not visible in the classical Lagrangian and appears only when the theory flows in the IR to an interacting superconformal fixed point.

The reader may have noticed that the theories $\mathrm{USp}(2k)^2$ and $\mathrm{U}(2k)$ with antisymmetric tensors can be interpreted as the world-volume theory of $\mathrm{SO}(2n)$ small instantons sitting at a Z_2 singularity [24, 25, 17]. The representation of these small instanton theories as systems of NS-, Dp- and Dq-branes was extensively discussed in [17, 18]; a T-duality along x_6 transforms the NS-branes into a space-time singularity, and the configuration of D-branes into a system of D2- and D6-branes, which have a natural interpretation as small instanton theories. In this approach the orientifold-invariant dispositions of two NS-branes on a segment accounts for all the possible breaking of $\mathrm{SO}(2n)$ to subgroups and the type of allowed bundles (with or without vector structure) [17]. Using the discussion in section 3, it is not difficult to see that also the mirror theories have the natural interpretation, in the spirit of [17], as the world-volume theory of $\mathrm{SU}(2)$ small instantons sitting at a D_n singularity. Once again, modulo minor differences due to the presence of ON^0, the disposition of the NS-branes on the segment accounts for the allowed global symmetry bundle. We see that the gauge group of the instanton theory and the group associated to the singularity are exchanged by mirror symmetry. This is not a fortuitous coincidence.

In [12] mirror symmetry was studied by realizing the gauge theories with configurations of D2- and D6-branes, and lifting them to systems of membranes sitting at orbifold singularities in M theory. This approach can be useful for studying the N=4 three-dimensional superconformal fixed points using the tools of the AdS/CFT correspondence [32, 33]. The compact examples considered in the previous sections can be reduced to configurations of D2- and D6-branes by performing a T-duality along x_6 and using the results of section 3. Consider only the case in which there is the same number of D3-branes everywhere; more general configurations are associated with fractional branes sitting at orbifold singularities. We obtain systems of D2-branes sitting at a Γ_G singularity in the presence of D6-branes realizing a G' global symmetry, where G and G' are $\mathrm{SU}(n)$ or $\mathrm{SO}(2n)$; these theories are the world-volume theories of small G' instantons sitting at a G singularity. Mirror symmetry correspond to the exchange of G with G'. The system can be lifted to a set of membranes in M-theory with a $G \times G'$ singularity; mirror symmetry is then reduced to the geometrical symmetry that exchanges G with G'. The argument is general and simple; unfortunately, the three-dimensional theory is not completely specified until we specify the form of the G' bundle on the Γ_G ALE space, since in general the global symmetries G and G' are broken to subgroups by the choice of a bundle. The case analyzed in the previous sections corresponds to the singularity $Z_2 \times D_n$.

The mutual disposition of NS- and D5-branes in both the original and the mirror theory determines a particular bundle, with a given global symmetry; in the case of the D-type quiver diagram, the disposition of the external lines gives a pictorial description of this bundle. The Type IIB description is useful for describing the most general bundle, and, more than that, provides a step-by-step method for computing the bundle for the mirror theory once the original one is given. Moreover, the type IIB description can be easily used also for configurations containing fractional branes.

Finally, we notice that interesting effects are associated with the singular cases of D_2 and D_3 singularities. In this paper we do not discuss these cases; however, they are easily realized using the methods discussed in this Section. The case of a D_2 singularity is particularly interesting since it provides examples of theories that are dual (in the sense that they flow to the same IR fixed point) at specific points in moduli space [34, 35]. The reason is that a D_2 singularity splits into two A_1 singularities. The M-theory lift of two D6-branes near an orientifold is the space $(\mathbb{R}^3 \times S^1)/\mathbb{Z}_2$. The two fixed points of $\mathbb{Z}_2$ correspond, in general, to different superconformal fixed points and the three-dimensional gauge theory may flow to one or the other according to the vacuum expectation value of the scalar parameterizing the position in S^1. It is not immediately obvious how these effects can be seen in the Type IIB description we are using in this paper.

4.3 The case with $Z_m \times D_n$ global symmetry

In this section we consider examples with global symmetry of type $Z_m \times D_n$. In the original configurations of branes constructed with orientifolds, these models are obtained by adding more NS-branes to the examples considered in section 4.1; in terms of the mirror theories, they are obtained by adding external lines to the D_n quiver diagram.

Consider only compact configurations. Non compact ones, which end with a NS-brane, can be extended to compact one by adding a second orientifold plane. As it should be clear from the examples in section 4.1, the mirror transform of a non-compact model can be obtained by the mirror transform of the corresponding compact one, by substituting the right part of the quiver diagram with a standard pattern of nodes associated to groups with decreasing rank, as depicted in figure 12.

We consider only, for simplicity, configurations with the same number of D3-branes everywhere. These are the kind of configurations that have a known T-dual description. The reader has all the elements for analyzing more complicated examples.

Consider figure 16*a*. m NS-branes can be put on the segment in an orientifold-invariant fashion in several ways [17, 18]. If m is odd, there must be a stuck NS-brane; each of the $[m/2]$ physical branes living not at the boundary of the segment has an image under the orientifold projection. If m is even, we can put $m/2$ physical branes

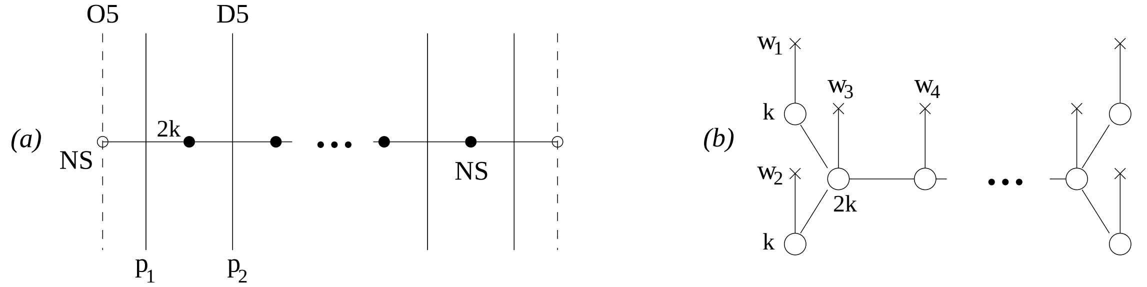

Figure 16: In figure (a) there are a total of $\sum_i p_i = n$ D5-branes and a total of m NS branes (carrying $m/2$ magnetic NS charge). The NS-branes are put on the segment in an orientifold-invariant way. Stuck NS-branes on the orientifolds can be present or not; this has been denoted using unfilled circles. The theory is $\mathrm{USp}(k) \times \mathrm{U}(k)^{m/2-1} \times \mathrm{USp}(k)$ with bi-fundamentals and n fundamentals distributed among the gauge factors, or analogous theories in which one or both the $\mathrm{USp}(k)$ factors are replaced by $\mathrm{U}(k)$ with an antisymmetric tensor, if there are one or two stuck NS-branes, respectively. The mirror theory is depicted in figure (b); the D_n quiver diagram has a maximum number m of external lines.

on the segment far from the boundary (case with vector structure) or we can put two stuck and $m/2-1$ physical NS-branes (case without vector structure). The theory is $\mathrm{USp}(k) \times \mathrm{U}(k)^{m/2-1} \times \mathrm{USp}(k)$ with bi-fundamentals and n fundamentals distributed among the gauge factors, or analogous theories in which one or both the $\mathrm{USp}(k)$ factors are replaced by $\mathrm{U}(k)$ with an antisymmetric tensor; the replacement occurs if there are one or two stuck NS-branes, respectively. These theories represent small $\mathrm{SO}(2n)$ instantons sitting at a Z_m singularity [24, 25, 17].

The mirror is a D_n quiver theory with an arbitrary distribution of a maximum of m fundamentals among the nodes. The exact mirror can be easily derived using the rules discussed in section 4.1.

These pairs of mirror theories are associated to M-theory singularities of the form $Z_m \times D_n$. These theories have been also considered in [12] in the simplified case in which $k=1$ and the maximal global symmetry is not broken.[9] The discussion in this section provides a general method for studying generic bundles and for determining the particular theory that corresponds to a generic diagram with a given distribution of external lines among the nodes.

The discrepancy between the number of parameters of the theories and of their mirrors mentioned in section 4.2 is even increased for $k > 2$. Some FI terms are missing; this was also noticed in [12].

[9]The proposal in [12], when compared to the one in this section, has an interchange between theories with vector structure and theories without. This happened since, in those days, there was no explicit method for computing the mirror theories in the case of D_n global symmetries, and the result was guessed on the basis of the M-theory intuition, the symmetry of the problem and the counting of parameters. In this paper we presented a method for explicitly computing the mirror theory.

4.4 The $D_n \times D_k$ global symmetry and self-dual models

We finish our discussion about mirror symmetry by considering the class of theories realized using both ON^0 and orientifolds planes. Consider, for example, figure 7. The three dimensional gauge theories that can be realized using these configurations are encoded in the quiver diagrams shown in the figure. We may ask: what is the strong coupling limit of these configurations? Luckily enough, the answer is very simple; since ON^0 and $O5^0$ are exchanged by S-duality, the mirror configuration has a form similar to the original one. It is encoded in a quiver diagram of the same form, with different numbers w_i and different dispositions of external lines. Consider only, for simplicity, compact models and configurations with the same number of D3-branes everywhere.

The maximally allowed global symmetry is $\mathrm{SO}(2n) \times \mathrm{SO}(2k)$, where $k = \sum w_i$. The disposition of external lines breaks this global symmetry to a subgroup $\mathrm{SO}(w_1) \times \mathrm{SO}(w_2) \times \mathrm{USp}(w_3) \times \mathrm{USp}(w_4) \times \cdots$. This is the world-volume theory of an $\mathrm{SO}(2k)$ small instanton sitting at a D_n singularity. The corresponding M-theory singularity is $D_n \times D_k$. Using the explicit S-duality, with the rules explained in the previous sections, or using the arguments in [12], we conclude that the mirror theory has the form in figure 17, but with k and n interchanged. The precise number and disposition of external lines can be determined by an explicit (and easy) computation.

It is natural to consider self-dual examples. If $k = n$, choosing suitable values for w_i, we easily obtain self-dual theories. In other words, we can consider these configurations as obtained by the orbifold generated by $\{(-1)^{F_L} R_{6789}, \Omega R_{3456}\}$; this is the meaning of having both ON^0 and $O5^0$, as discussed in section 2.2. Under an S-duality, Ω and $(-1)^{F_L}$ are interchanged. The orbifold is therefore unaffected by an S-duality combined with the interchange of (345) with (789) (this are the rules of the game [11]!); configurations in which the total D5-charge and the total NS-charge, as well as their disposition in space-time, are the same (this certainly implies $k = n$) are therefore expected to be self-dual.

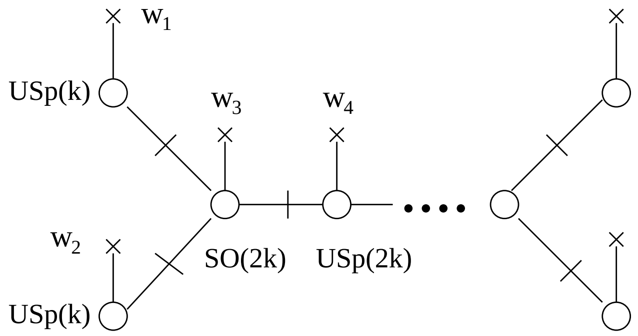

Figure 17: A D_n quiver diagram with $k = \sum w_i$ external lines. It corresponds to a global symmetry associated to $D_n \times D_k$. The mirror theory has the same form, with k and n interchanged and with a different partition of $n = \sum \tilde{w}_i$ external lines.

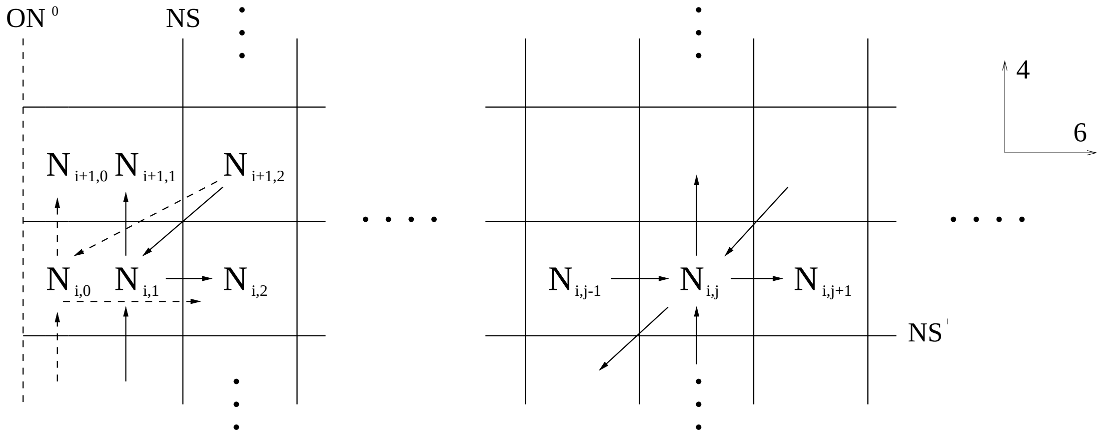

Figure 18: A generic Brane Box Model with ON^0 planes. The matter fields in the fundamental or anti-fundamental representation, which are represented as oriented arrows, are indicated for two particular boxes. The boxes near ON^0 are split into two U(N) gauge factors. The matter fields for U($N_{i,1}$) are indicated as standard arrows, while those for U($N_{i,0}$) are indicated as dashed arrows. If $N_{i,j} = 2N$ for $j \neq 0, 1$ and $N_{i,0} = N_{i,1} = N$, there is no bending and the one-loop beta function is zero. Under the same condition, if the model is compact in both the horizontal and vertical direction, the theory is finite and related under T-duality to D3-branes near an orbifold of C^3 generated by Z_k and D_n, where k is the number of NS$'$-branes and $n - 2$ is the number of NS-branes.

4.5 Open problems

There are open questions which are not solved by the current methods we use. There are two basic unsolved questions.

The mirror of SO gauge theories and their generalizations. We refer to configurations which involve $O3^+$ or $O5^+$ planes. It is not clear at the moment how to understand the mirror of these planes and this is the source for the confusion.

There are two basic constructions of USp gauge theories using D3 branes, D5 branes and NS branes. One construction uses an $O3^+$ plane and the other one an $O5^-$ plane. In both cases the resulting gauge group is USp. Correspondingly, the Type IIB S-dual of both configurations is expected to give the known dual. Currently only one configuration gives a satisfactory answer. The one with an $O5^-$ plane. The configuration with $O3^+$ is not understood well enough to produce the right mirror.

5. Four dimensional theories

5.1 Brane box models

It is almost straightforward to introduce an ON^0 plane in the Brane Box Models studied in [13]. Consider figure 18.[10] There are D5-branes extending in (012346),

[10] An interesting developement which generalizes Brane Box Models is given in [36] and is termed Brane Diamonds. It would interesting to generalize the contents of this section along these lines.

NS-branes and ON^0 planes extending in (012345) and NS$'$-branes extending in (012367). The model has $N = 1$ supersymmetry and is generically chiral. The generic box $N_{i,j}$ gives rise to the gauge group $\mathrm{U}(N_{i,j})$. The open strings connecting neighboring boxes give rise to chiral matter fields in the fundamental (or anti-fundamental) representation of the neighboring gauge groups. These matter fields and their chirality are depicted using oriented arrows. Only arrows directed and oriented East, North and South-West exist [13]. All the existing matter fields are indicated for the box $N_{i,j}$. Every time three arrows close a triangle there is a cubic superpotential [13]. The only novelty regards the boxes near ON^0, for example $N_{i,0}$ and $N_{i,1}$. As we can expect from section 2, there is a splitting into two gauge factors $\mathrm{U}(N_{i,0})$ and $\mathrm{U}(N_{i,1})$. The action of $(-1)^{F_L}$ on the Chan-Paton factors is a diagonal matrix with entries $+1$ for the indices associated to $N_{i,0}$ and -1 for those associated to $N_{i,1}$. As a consequence, the open strings connecting, say, $N_{i,0}$ with $N_{i\pm1,1}$ are projected out. The resulting spectrum is indicated in figure 18 for the box $N_{i,1}$ and $N_{i,0}$; the standard arrows represent the matter fields for the gauge group $\mathrm{U}(N_{i,1})$, while the dashed arrows represent the matter fields for $\mathrm{U}(N_{i,0})$.

The numbers $N_{i,j}$ must be chosen in order to have an anomaly-free model. Anomalies on the world-volume of the branes should be related to the violation of the equations of motion for some space-time field. Due to the complexity of the model and the non-trivial bending of the NS and NS$'$-branes, the precise relation between anomalies and charge conservation of the string background is not known. Attempts to find such a relation appeared in the literature [37, 38, 39, 40] without a conclusive result. In principle, it may happen that, even if the number N_{ij} are such that they cancel anomalies, the branes background is inconsistent. However, the consistency of a large class of Brane Box Models can be explicitly checked by T dualizing them to other consistent systems of branes sitting at orbifold singularities [15, 39]. The class of consistent models can be enlarged by considering models which are separately well defined and sewing them [37, 38, 39, 40].

There is an obvious example that is well defined. If we take the same number of D5-branes in each box there is no bending for the NS-branes and the space-time equations of motion are satisfied. This corresponds to $N_{i,j} = 2N$ for $j \neq 0, 1$ and $N_{i,0} = N_{i,1} = N$. The condition of no bending is equivalent to vanishing of the total charge of the D5-branes ending on a given NS and NS$'$-branes. The analogous condition for the ON^0 plane forces us to take $N_{i,0} = N_{i,1}$. Since the bending is associated to the running of the coupling constant [41], this model has zero one-loop beta function [14]. Every gauge group $\mathrm{U}(n)$, including those realized near ON^0, has indeed $3n$ fields in the fundamental and $3n$ in the anti-fundamental representation. The quantum field theory is conjectured to be finite using the same argument of non-bending as in [14].

5.2 Relation to branes at orbifold singularities and finite models

We can easily construct cylindrical (6 direction compact) and elliptical (6 and 4 directions compact) models. These kind of models can be related by T-duality to systems of branes at orbifold singularities [13, 14, 15]. Consider $n-2$ NS-branes and k NS$'$-branes. According to the discussion in section 3, we expect that the NS$'$-branes are replaced by a T-duality along the 4 direction with an A-type singularity and the NS-branes and the ON^0 planes are replaced by a T-duality along the 6 direction with a D-type singularity. The cylindrical models are the T dual of systems of D4 and NS-branes at a D_n orbifold singularity extended in the directions $6,7,8,9$ [13, 42]. The elliptical models are the T dual of systems of D3-branes sitting at a C^3 orbifold; the actual orbifold group is the smallest discrete group acting on C^3 that contains a Z_k subgroup acting on the coordinates $4,5,8,9$ and a D_n subgroup acting on the coordinates $6,7,8,9$. To preserve $N=1$ supersymmetry, this discrete group must be a subgroup of SU(3) [43]. It would be quite interesting to further analyze the relation between the Brane Box Models and the construction in [43].

When $N_{i,j}=2N$ for $j\neq 0,1,n,n-1$ and $N_{i,0}=N_{i,1}=N_{i,n-1}=N_{i,n}=N$, the D3-branes transforms in the regular representation of the orbifold group [19]. However, in general, the numbers $N_{i,j}$ do not need to be all equal. We may project the D3-branes with a different representation. This would result in having fractional D3-branes. The anomaly cancellation becomes now equivalent to the tadpole cancellation [39].

We may obtain more complicated orbifold singularities by compactifying in the 6 direction allowing for a shift along the 4 direction [15]. The shift must be compatible with the Z_2 projection induced by ON^0. It appears that the Brane Box Models give a simple construction of the gauge theory and the superpotential of some of the theory associated with discrete groups of SU(3). It would be interesting to analyze the dictionary for translating these kind of models into an orbifold projection, along the lines of [15, 43].

It is believed that the properties of finiteness improve if the models are compact [14, 15]. If we take $N_{i,j}=2N$ for $j\neq 0,1,n,n-1$ and $N_{i,0}=N_{i,1}=N_{i,n-1}=N_{i,n}=N$ and compactify along the 6 direction, allowing for some shift, we obtain a finite and conformal $N=1$ model. As we said, this is mapped by T-duality to a set of D3-branes sitting at an orbifold singularity. The finiteness of this model then follows from the AdS/CFT correspondence using the arguments in [44].

6. Six-dimensional theories

In this section we discuss examples of six-dimensional gauge theories.

There are several consistent string backgrounds that give rise to anomaly-free six-dimensional theories [45, 46, 47, 3, 26, 24, 25]. In [17] we described a realization with D6-, D8- and NS-branes. The introduction of an ON^0 plane in the original construction in [16, 17, 18] allows to realize more complicated examples. This section completes the results in [17]. It also shows how to construct (in the spirit of [11]) the gauge theories associated to small SO(32) instantons sitting at D_n singularities.

In section 2, we discuss in detail what is the world-volume theory of Dp-branes ending on NS-branes and ON^0 planes. In the case of D6-branes, we must include the fields living on NS-branes or ON^0 planes in the six-dimensional theory and we must also pay attention to the anomaly cancellation conditions. As a general rule, an anomaly in field theory translates in the brane set-up to the non-conservation of some charge for bulk fields. In this particular case, the relevant charges are associated to the fields living on the NS-branes; for a remarkable return, the same fields provide the tensor multiplets that are necessary to completely cancel the anomalies in six-dimensions.

The general discussion about anomaly cancellation in the brane set-up can be found in [48, 17]. The general rule will be clear after considering a specific example. The six-dimensional theory corresponding to figure 3b is still $\mathrm{U}(n_1) \times \mathrm{U}(n_2)$ without bi-fundamentals, but now with two tensor multiplets and two hyper-multiplets coming from the NS-brane and the ON^0 plane. Naively, one would think that only one linear combination of these multiplets is relevant for the field theory, arguing that the sum of the two multiplets decouples while the difference appears as a gauge coupling and FI parameter, respectively, for both gauge groups. However, this is not true. The two tensor multiplets and two hypermultiplets couple both to the gauge theory. The sum of the multiplets couples to, say, the gauge group on the D6 branes which have positive charged with respect to the ON^0 multiplet while the difference of the multiplets couples to the gauge group associated with the negatively charged D6 branes. The theory in figure 3 is generally anomalous. A $\mathrm{U}(N)$ theory with N_f flavors indeed is anomalous unless $N_f = 2N$ and the theory is coupled to a tensor multiplet, the scalar of which plays the role of a gauge coupling. It is easy to see what is wrong with charge conservation for bulk fields. Since the world-volume of the D6-brane is bigger than that of a NS-brane, the RR-charge of the D6-brane can not be absorbed by the NS-brane as it happens for all the Dp-branes ending on a NS-brane for $p \leq 5$. Therefore the D6 charge must be canceled locally at the position of the NS-brane. In this case, there are $n_1 + n_2$ D6-branes on the left of the NS-brane and zero on the right and the charge is not conserved.[11]

[11]Notice that, while a Dp-brane may have positive or negative charge under the fields living on ON^0 [9], the charge under the NS-brane fields is always positive for branes ending, say, on the left. We are not considering anti Dp-branes in our picture; they would break supersymmetry. Actually, one can interpret Sen's assignment of positive and negative charges as being ending 'to the left' or 'to the right' of the ON^0 plane, respectively as in figure 2.

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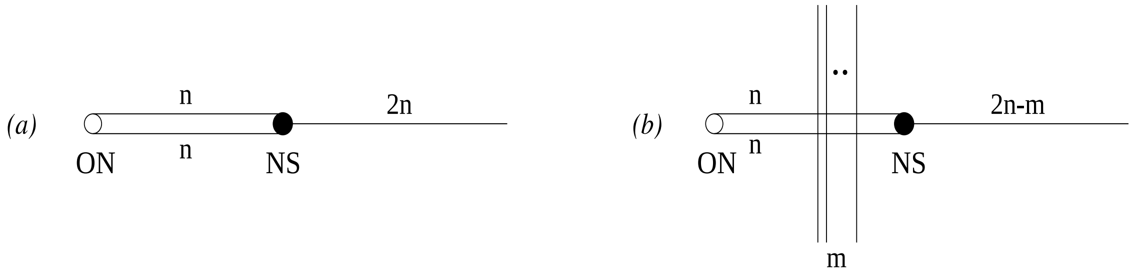

Figure 19: The standard figure for a six-dimensional theory. D6-branes are depicted as horizontal lines ending on points corresponding to NS-branes (filled circles) and ON^0 planes (empty circles). D8-branes are depicted as vertical lines.

There are two ways of removing the obstacle and constructing an anomaly-free theory. They are depicted in figure 19. Notations are changed with respect to figure 3. Since the D6-worldvolume is bigger than the NS one, it is better to represent D6-branes as lines ending on points which represent NS-branes. In figure 19*a*, we added $n_1 + n_2$ semi-infinite D6-branes to the right of the NS-brane in order to preserve the charge. The two kinds of D6-branes ending on ON^0 have opposite charge with respect to the fields living on it and we must cancel the ON^0 tadpole by taking $n_1 = n_2$. The resulting theory is $\mathrm{U}(n) \times \mathrm{U}(n)$ with $2n$ flavors for each group, two tensor multiplets and two hypermultiplets. The tensor multiplets are necessary to cancel the non-Abelian anomalies [49] while the hypermultiplets, acting as dynamical FI terms, cancel the Abelian anomalies, making the U(1) factors massive [19, 22]. In figure 19*b*, we also added D8-branes, depicted as vertical lines. They induce a cosmological constant, which is constant in space-time and jumps by one unit when one crosses a D8-brane [50]. A non-zero cosmological constant m induces an effective RR seven form charge at the position of a NS-brane. This changes the charge conservation condition in the following way: the D6 charge on the left of a NS-brane minus the D6 charge on the right must equal the value of the cosmological constant at the position of the NS-brane:[12] $n_l - n_r = m$ [48, 17]. The cosmological constant is zero near ON^0 implying again $n_1 = n_2$. It is instead equal to m near the NS-brane implying that we must add $2n - m$ semi-infinite D6-branes to its right. The resulting theory is again $\mathrm{U}(n) \times \mathrm{U}(n)$ with $2n$ flavors for both groups, two tensor multiplets and two hypermultiplets.

Equipped with the rules discussed in section 2, we can construct more general examples. Adding NS-branes to the initial ON^0 plane, without including orientifolds, we obtain theories that are products of $\mathrm{U}(N)$ gauge groups. This kind of examples are exhaustively discussed in [17].

[12] We are using conventions in which a physical D6-brane counts $+1$ and a physical D8-brane induces a cosmological constant of magnitude $+1$.

Introducing an $O8$ plane, we can construct many more examples, containing, in particular, the configurations T dual to SO(w) small instantons on D_n ALE spaces.

Compact model with two ON^0 planes are associated to small instanton theories, as discussed in section 3. In the six-dimensional case, we must pay attention to the anomaly cancellation, which usually constrains the world-volume theory. Theories of small U(N) or SO($2N$) instantons living at a D_n singularity are associated with a D_n extended Dynkin diagrams as in figure 9 with the addition of external lines as in figure 4, for U(N), or figure 7, for SO($2N$). The n tensors and hypermultiplets required to cancel both non-Abelian and Abelian anomalies are automatically provided by the $n-2$ NS-branes and the two ON^0 planes.

The introduction of an $O8^-$ plane affects our picture much more than in the original examples in [17]. The general strategy in [17] was to replace every space-time singularity with a dual background where the singularity was replaced by branes. This in general allows to have a better control on the parameters and moduli of the theory. In the case of D_n singularities, we will have a mixed picture, with both NS-branes and the perturbative orbifold projection associated to ON^0 plane.

The generic quiver theory in the presence of $O8^-$ plane was already discussed, for arbitrary dimension, in section 2.2 (figure 7). The form of the world-volume theory does not depend on the dimension. What is peculiar to six-dimensions is that the anomaly cancellation uniquely determines the world-volume theory. Moreover we need to include fields from the NS-branes and from the twisted sectors in the theory; they are crucial for canceling the anomalies. Recall from section 2.2, that there are two consistent configurations of D-branes living at the intersection of an $O8^-$ and an ON^0 plane. The first one has an $O6^-$ plane, no splitting of branes, U-type gauge groups and an hypermultiplet from the twisted states. The second one has an $O6^+$ plane, a splitting with USp-type gauge groups and a tensor multiplet from the twisted states. In six-dimensions, it is crucial to keep track of the twisted sectors, since they contribute to the theory and are important in canceling the anomalies.

The actual gauge groups (the numbers $k, r, t, s, \ldots$ in figure 7) can be determined by the RR-charge conservation condition, which is equivalent to the anomaly cancellation [17]. We recall that we are using notations where $O8^-$ contributes -8 to the cosmological constant and $O6^\pm$ has RR-charge ± 4. The only new ingredient, not present in [17], is the condition that must hold near ON^0. The total charge for the ON^0 twisted tensor field must be canceled, since there is no room for the D6-brane RR-charge to escape. This fixes the splitting (the numbers k and r) near ON^0. The rule is that $k - r = w_1$. It is easily checked that it is equivalent to the anomaly cancellation condition; we give an explicit example below. This result would surely follow from a careful analysis in terms of boundary states, along the lines of [9]. We prefer to give a heuristic argument in the spirit of the S-dual picture of figure 2. It is convenient to represent the ON^0

plane as splitted into a fixed plane and a *virtual* NS-brane which supports the twisted fields (see figure 20). The w_1 Dq-branes now live on the left of the virtual NS-brane. We assume that the virtual NS-brane behaves at all effects as a real NS-brane. This means that the orientifold plane $O6$ change sign in crossing it. More important, the anomaly cancellation condition for the ON^0 plane is mapped to the standard condition for a NS-brane. The cosmological constant at the virtual NS-brane is $-8+w_1$, where -8 is the contribution of $O8^-$. The condition is $(2k-4)-(r+k+4)=-8+w_1$, which is the same as $k-r=w_1$; the ±4 are the contribution of the $O6$ plane on the right and on the left of the virtual NS-brane, respectively.

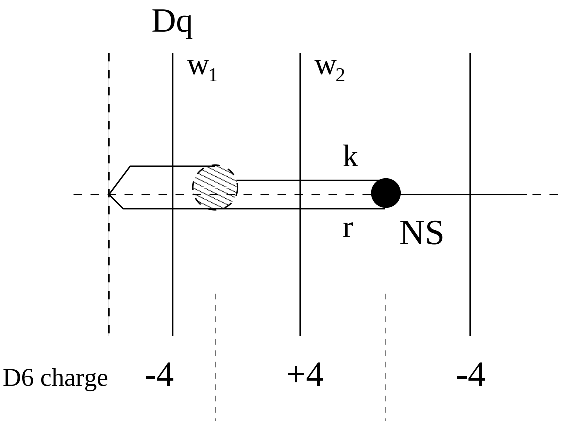

Figure 20: The anomaly cancellation rule near ON^0. ON^0 is represented as the pair containing a fixed plane and a virtual NS-brane.

It is quite easy to construct compact models by putting another $O8^-$ plane in the six direction and taking $\sum w_i = 16$. This will describe the T dual version of SO(32) small instantons sitting at D_n singularities [24, 25, 51]. The theory will change according to where we add the $O8$ plane. If n is even,[13] $O8$ will intersect an $O6^+$; therefore we will need the prescription in figure 5b. If n is odd, $O8$ will intersect an $O6^-$ plane and we will need figure 5a. The resulting theory is the same as the one discussed in [24, 25, 51]. In figure 21 we depicted the theory associated with small instantons for D_4 and D_5. The general theory for arbitrary n should be obvious from these examples; it is associated to an extended (affine) Dynkin diagram of D-type for n even, and to a standard (without the extra node) Dynkin diagram of D-type for n odd.[14] Charge conservation fixes the gauge groups in such a way to cancel all the anomalies. As an example of the application of the previous rules, we consider the D_4 example in figure 21.

We put $w_1 = 16$ and all the other $w_i = 0$. There is a total of four conditions.

1. Near the first ON^0; the previously discussed condition is $2k-2r=w_1=16$.

2. Near the first NS-brane. $O6$ contributes a $+4$ RR-charge on the left and -4 on the right. The cosmological constant is $-8+w_1=8$, where -8 is the contribution of $O8^-$. The condition is $(2k+2r+4)-(2t-4)=8$.

[13] We know from section 1 that for the description of a D_n singularity we need $n-2$ NS-branes and two ON^0 planes.

[14] To be precise, if we consider the geometrical construction of these theories in [24, 25], we realize that, for n odd, it is not the extended node of the diagram that is missing, but the last two nodes (the $n-1$th and nth) are identified.

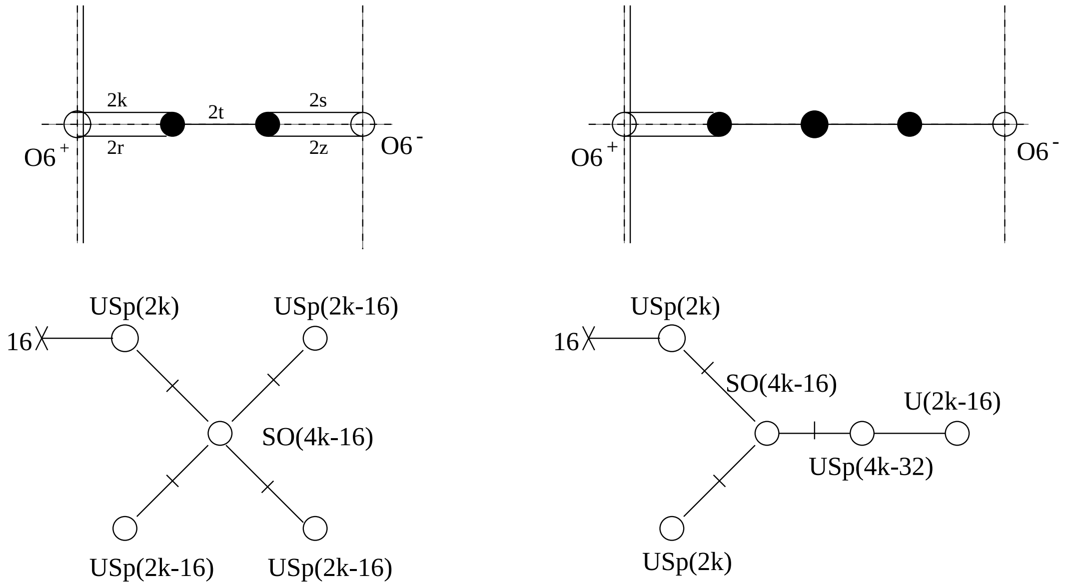

Figure 21: The SO(32) small instanton theory for D_4 and D_5 singularities. For simplicity, we considered an SO(32) unbroken group, obtained by putting all the 16 D8-branes at the same point, near ON^0. The case for general breaking of the global symmetry group follows by partitioning the D8-branes in between the NS-branes. The charge conservation rules described in the text uniquely fix the world-volume theory. The generalization to an arbitrary D_k is obvious.

3. Near the second NS-brane: $(2t-4)-(2s+2z+4)=8$.

4. Near the last ON^0, where there are no D8-branes: $2s-2z=0$.

This set of four equations determines the world-volume content indicated in figure 21. The theory is anomaly free.[15] For n even, the theory contains n tensor multiplets ($n-2$ NS-branes and two ON^0 planes intersecting $O6^+$ planes); for n odd, $n-1$ tensor multiplets and one FI hypermultiplet (ON^0 intersecting $O6^-$). This is exactly what is required to cancel the Abelian and non-Abelian anomalies.

6.1 Generalizations

We can easily construct generalizations of the small instanton theories and build new models. A first example is easily obtained by constructing theories in which the ON^0 plane always intersects an $O6^-$ plane. There is no splitting of branes near ON^0 and the quiver diagram is a simple line of nodes. This configuration is allowed only if n

[15] Recall that a USp(N_c) theory is anomaly free if $N_f = N_c + 8$, while an SO(N_c) theory is anomaly free if $N_f = N_c - 8$. In both cases, a tensor multiplet is needed to completely cancel the anomaly.

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is even. The world-volume theories are of the form,

$$\begin{aligned} &\mathrm{U}(k_1) \times \mathrm{USp}(k_2) \times \mathrm{SO}(k_3) \times \cdots \\ &\qquad \cdots \times \mathrm{USp}(k_{n-2}) \times \mathrm{U}(k_{n-1}) \,, \end{aligned} \tag{6.1}$$

with bi-fundamentals or half bi-fundamentals for neighboring factors and fundamentals for the various gauge groups. This model was already considered in [17]. The numbers k_i are easily determined using the rules found in the previous section. After orientifold projection, the two ON^0 planes provide the two FI hypermultiplet needed for canceling the Abelian anomalies and the NS-branes provide the $n-2$ tensor multiplets needed for the non-Abelian anomalies. We can speculate that these theories correspond to a geometrical SO bundle at a D_n singularity without vector structure, which indeed exists only for n even. We did not find the explicit construction of theories without vector structure for D-singularities in the literature, but it would not be too difficult to construct them using the methods in [24, 25]. In figure 22 we depicted the D_4 example. The generalization to D_n is straightforward.

A second obvious generalization involves the introduction of an $O8^+$ plane. It has charge $+8$, therefore if we have both $O8^-$ and $O8^+$ in a compact model there is no need for D8-branes. The discussion about the properties of $O8-ON^0$ planes is invariant under the simultaneous change of sign of the charge of the $O8$ and $O6$ planes. This means that there are two consistent configurations of D-branes living at the intersection of an $O8^+$ and an ON^0 plane. The first one has an $O6^+$ plane, no splitting of branes, U-type gauge groups and a hypermultiplet from the twisted states. The second one has an $O6^-$ plane, a splitting with SO-type gauge groups and a tensor multiplet from the twisted states. In the case of a D_n singularity, we obtain standard D-type Dynkin diagrams for n odd, and extended (affine) D-type Dynkin diagrams for n even. This is the opposite of what we found when the two $O8$ planes had both the same negative charge. The D_4 and D_5 cases are depicted in figure 23.

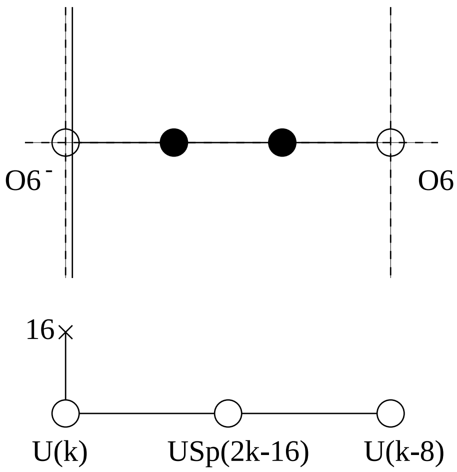

Figure 22: A small instanton without vector structure sitting at a D_4 singularity.

We discussed several examples. They are complete in the sense that they cover all the possible configurations which contain ON, $O8$ and $O6$ planes. By combining in a different way the various players of this game, we can construct many other compact and non-compact models. The rules we gave in the previous section for determining the world-volume theory will always produce a consistent anomaly-free six-dimensional theory with a non-trivial fixed point. We will not discuss the appli-

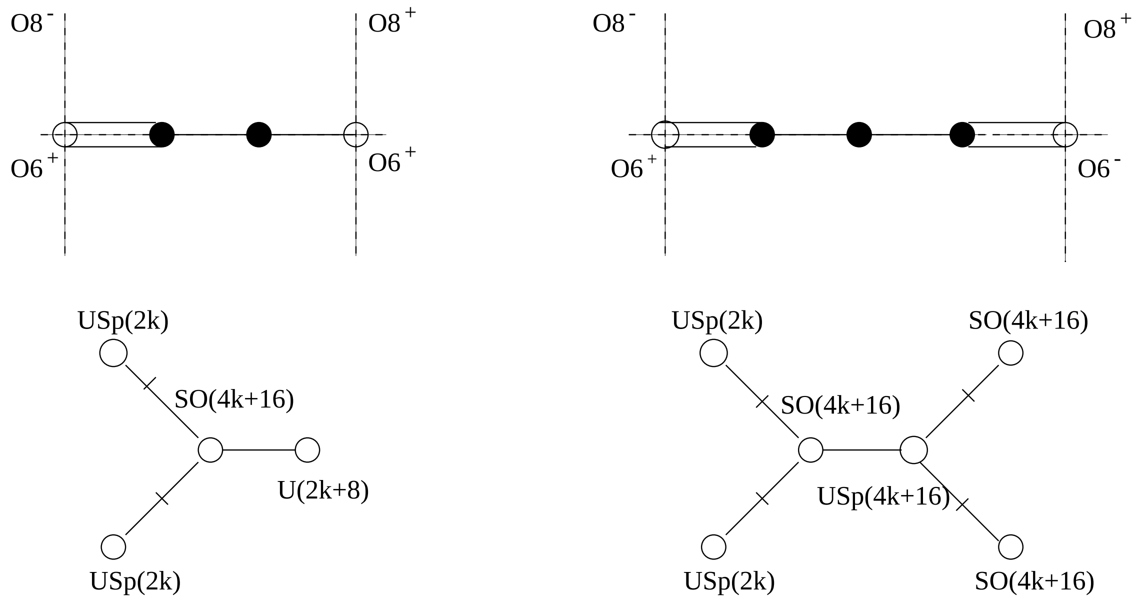

Figure 23: D_4 and D_5 theories constructed with the use of $O8^+$.

cations of what we said to all the rich aspects of the six-dimensional physics, since the general philosophy of the brane construction for six dimensional theories was already presented in [17].

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